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The Motion of Coaxial Bodies of Varying Composition on the Active Leg of Descent

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Abstract—The motion of a spacecraft (SC) with double rotation and variable mass on the active leg of its descent is considered. The SC consists of two coaxial bodies. The coaxial scheme is used for gyroscopic stabilization of the SC longitudinal axis by the method of partial spin-up. The equations of spatial motion of coaxial bodies of varying composition are derived and approximate solutions for the angles of spatial orientation are found. The condition of decreasing amplitude of nutation oscillations is obtained, which allows the estimation of efficiency of the stabilization by partial spin-up. The errors in the magnitude and direction of the vector of braking thrust are also determined.

1. INTRODUCTION

When an SC descends in the atmosphere, it receives a decelerating impulse formed by the de-orbiting brake engine which transfers the SC from its initial orbit to the descending one. When the propellant in the decelerating engine burns out, the inertia–mass parameters of the SC vary. The braking thrust vector should have a prescribed magnitude and direction in space in order to guarantee the condition of the SC transfer to the required orbit of descent. As a rule, in order to do that, gyroscopic stabilization of the SC longitudinal axis, with which the braking thrust vector is related, is used. One of the methods of gyroscopic stabilization is the employment of the partial spin-up [1], when the stabilizing block—the brake engine which is separated after the propellant exhaustion and which carries away the stabilizing gyroscopic moment—is rotated, while the SC itself is not rotated, which allows one to increase the SC effective mass due to the refusal to employ devices of extinguishing the residual angular velocity.

The system of extinguishing the angular velocity of stabilization of small SC (with a mass of up to 65 kg and dimensions of up to 1 m) used in the domestic systems of remote sensing of the Earth's surface (RSES) and designed according to the scheme of a single solid body is represented by two loads with masses of 400 g on unwinding cables with a mass of 50 g fixed to the SC mainframe in the fastenings allowing the cables to extend at their total unwinding, which is known as the Yo-Yo system [2]. At the first place the use of coaxial SC allows one to increase the payload mass by 1 kg, which is an appreciable value in relation to the mass of an SC for RSEC itself. At the second place, a failure or malfunction in operation of the Yo-Yo may lead to a situation, when, though the angular momentum of stabilization exists, a statically stable SC will not be able to level along the stream in the atmosphere, which will result in a failure of the parachute system and, consequently, in the loss of SC. Thereby, the scheme of coaxial SC seems to be more reliable, the more so that in modern SC for RSES the brake engine is dropped off in any case, otherwise the hatch of the parachute system is blocked.

In the presence of perturbations the longitudinal axis of SC may execute nutation–precession motion with respect to the center of mass. Deviations of the longitudinal axis and, consequently, of the thrust vector lead to the transition to the orbit of descent differing from the required computed orbit and, as a result, to an increase of the region of scatter of landing points (Fig. 1).

We formulate a problem of constructing the equations of spatial motion of an SC which is a system of coaxial bodies of varying mass, of obtaining the

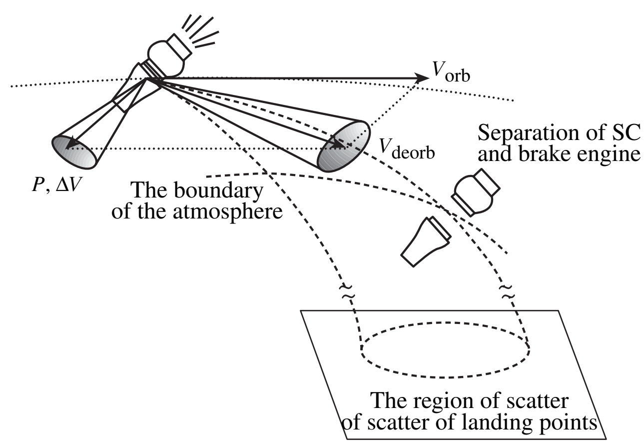

Fig. 1.

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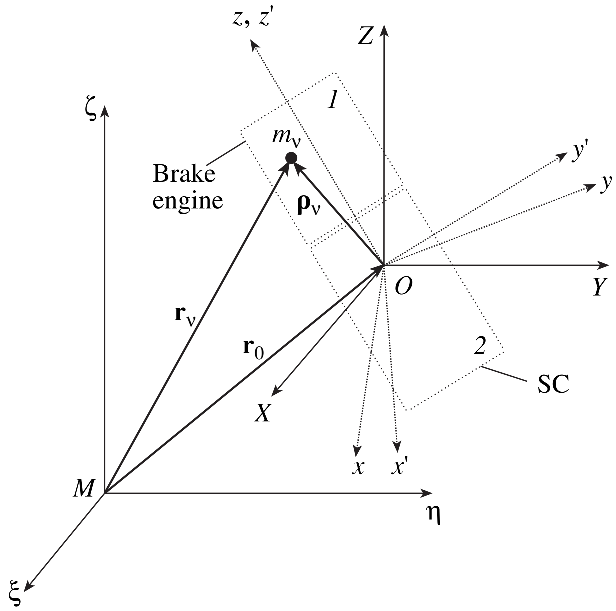

Fig. 2.

approximate solutions for the parameters of spatial orientation of the system on a time interval corresponding to the duration of operation of the de-orbiting brake engine, and of determining the constraints imposed on the inertial-mass parameters guaranteeing the smallest values of nutation oscillations and stabilization errors.

Let us present the main parameters related to this problem. As initial orbits from which the descent is realized one may use circular orbits with heights of 200 to 300 km or elliptic orbits with heights of 300 and 200 km in apogee and perigee, respectively, and with an inclination of 65°.

Let us call *the active leg of descent* a short interval of motion of SC of varying mass on which the brake engine operates and decelerating impulse is applied. The de-orbiting brake engine operates for 15–25 s, and during this time the complete burnout of solid propellant occurs.

It should be noted that in most problems of space flight dynamics related to the impulse interorbital maneuvers it is assumed that decelerating and accelerating impulses are instantaneous, their magnitudes and directions being specified [3]. However, as it was noted above, both variation of the thrust direction and variation of the impulse magnitude due to its scatter around the necessary direction may take place in the process of SC motion. In our formulation of the problem we consider more carefully the processes of generation of impulses on a short but finite time interval of brake engine operation taking into account the attitude motion of SC.

2. THE EQUATIONS OF MOTION OF COAXIAL BODIES

Describing the motion of the systems of varying composition let us use a hypothesis of a short-range interaction [4, 5] on which the method by I.V. Meshchersky is based. According to this hypothesis the particles are thrown off only from a certain part of the surface of the body of varying mass, and particles that have no relative velocity with respect to the body-fixed coordinate system are assumed to belong to the body, while particles having (received) such a relative velocity do not belong to the body and do not influence its motion. In this case, the reactive forces and moments are understood as a result of such a contact interaction of the thrown off particles and the body only at the instant of their separation from the main body.

The center of mass of the system moves with respect to SC in the process of propellant burning. Let us write the equations of motion in the coordinate system *Oxyz* rigidly fixed to SC and having the origin at point *O* coinciding with the initial position of the center of mass of the system of bodies.

Let us introduce the following coordinate systems (Fig. 2): $M\xi\eta\zeta$ is the inertial coordinate system; *OXYZ* is a mobile coordinate system whose axes remain colinear to the axes of the inertial system throughout the time of motion; *Oxyz* and *Ox'y'z'* are the coordinate systems fixed to SC (body *2*) and brake engine (body *1*), respectively, rotating with respect to the *OXYZ* coordinate system.

Let us write the expression of the theorem on variation of the angular momentum of a system with variable mass [4]:

$$\frac{d\mathbf{K}}{dt} = \mathbf{M}^e + \mathbf{M}^R + \sum_\nu \mathbf{r}_\nu \times \frac{dm_\nu}{dt}\mathbf{v}_\nu, \tag{2.1}$$

where $\mathbf{M}^e$ is the principal moment of external forces, $\mathbf{M}^R$ is the principal moment of reactive forces, and $\sum_\nu \mathbf{r}_\nu \times \frac{dm_\nu}{dt}\mathbf{v}_\nu$ is the sum of angular momenta of the particles thrown off in the unit of time in their translational motion with respect to the immobile coordinate system.

Equation (2.1) for the system of two coaxial bodies can be transformed following a scheme suggested in [4] to a form determining the theorem on variation of the angular momentum with respect to the *OXYZ* coordinate system [6]:

$$\sum_{i=1}^{2}\frac{d\mathbf{K}_{i,O}}{dt} - \sum_{i=1}^{2}\sum_{\nu_i}\boldsymbol{\rho}_{\nu_i}\times\frac{dm_{\nu_i}}{dt}(\boldsymbol{\omega}_i\times\boldsymbol{\rho}_{\nu_i}) = \mathbf{M}_O^e + \mathbf{M}_O^R - \boldsymbol{\rho}_C\times m\mathbf{w}_O. \tag{2.2}$$

Here, $\boldsymbol{\rho}_C$ is the radius vector of the center of mass of the entire system varying due to variation of the mass,

$\boldsymbol{\omega}_i$ and $\boldsymbol{\varepsilon}_i$ are the absolute angular velocities and accelerations of bodies i ($i = 1, 2$); $\mathbf{K}_{i,O}$ are the bodies' angular momenta calculated with respect to pole O; m is the mass of the entire system; $\mathbf{M}_O^e = \boldsymbol{\rho}_C \times \mathbf{F} + \mathbf{M}_C^e$ is the moment of external forces relative to point O written through the principal vector of external forces $\mathbf{F}$ and principal vector of external moments $\mathbf{M}_C^e$ with respect to the center of mass; and $\mathbf{w}_O$ is the absolute acceleration of pole O: $\mathbf{w}_O = (\mathbf{F} + \boldsymbol{\Phi}^R - \boldsymbol{\varepsilon}_2 \times m\boldsymbol{\rho}_C - m\boldsymbol{\omega}_2 \times \boldsymbol{\omega}_2 \times \boldsymbol{\rho}_C)/m$, where $\boldsymbol{\Phi}^R$ is the principal vector of reactive forces.

The right-hand side of relation (2.2) can be transformed to the form

$$\mathbf{M}_O^e + \mathbf{M}_O^R - \boldsymbol{\rho}_C \times m\mathbf{w}_O$$
$$= \mathbf{M}_C^e + \mathbf{M}_C^R + \boldsymbol{\rho}_C \times (\boldsymbol{\varepsilon}_2 \times m\boldsymbol{\rho}_C + m\boldsymbol{\omega}_2 \times \boldsymbol{\omega}_2 \times \boldsymbol{\rho}_C).$$

Let us consider the motion of SC, taking into account only gravitational forces and neglecting other external effects. In this case, for both flat and central gravitational fields, taking into account the smallness of SC dimensions with respect to the radius of the orbit, we may assume that the moment $\mathbf{M}_C^e$ vanishes. Let body *1* (brake engine) be a body of variable composition and body *2* (SC) be a body of constant composition. Let us consider the process of symmetric burning of propellant in the brake engine, when particles are thrown off strictly in the direction of longitudinal axis without linear and angular thrust eccentricities. Then in the process of variation of the composition of body *1*, dynamical symmetry is not violated, and moment $\mathbf{M}_C^R$ of reactive forces with respect to the center of mass is zero.

Let us write the angular velocities and angular momenta of bodies in projections onto the axes of their bound coordinate systems:

$$\boldsymbol{\omega}_1 = p'\mathbf{i}' + q'\mathbf{j}' + r'\mathbf{k}'; \quad \boldsymbol{\omega}_2 = p\mathbf{i} + q\mathbf{j} + r\mathbf{k};$$

$$\mathbf{K}_{1,O} = A_1(t)p'\mathbf{i}' + A_1(t)q'\mathbf{j}' + C_1(t)r'\mathbf{k}';$$

$$\mathbf{K}_{2,O} = A_2 p\mathbf{i} + A_2 q\mathbf{j} + C_2 r\mathbf{k},$$

where A_i and C_i are the equatorial and longitudinal inertia moments of body i calculated in the coordinate systems $Oxyz$ and $Ox'y'z'$ fixed to the bodies, and $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ and $\{\mathbf{i}', \mathbf{j}', \mathbf{k}'\}$ are the unit vectors of the considered coordinate systems.

Let us designate the angle and velocity of spin-up of body *1* with respect to body *2* in the direction of the longitudinal direction Oz as δ and σ, respectively, where $\sigma = \dot{\delta}$. The parameters of spatial orientation are shown in Fig. 3.

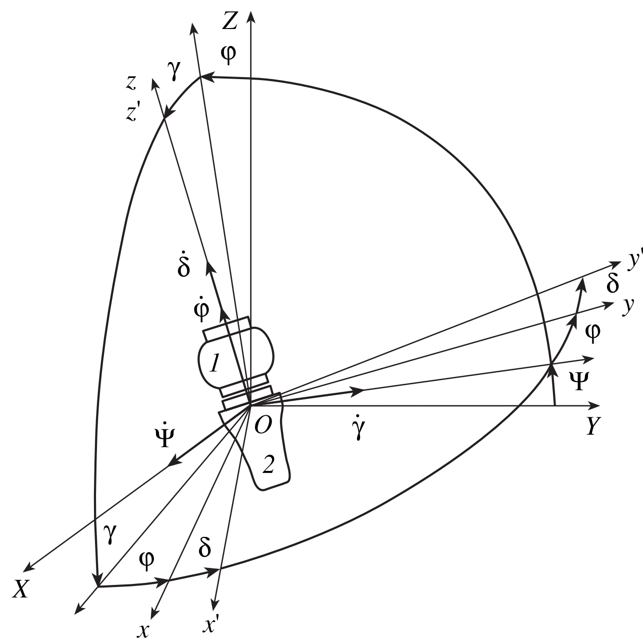

Fig. 3.

On the basis of vector equation (2.2) one can arrive at the following system of scalar dynamical equations of motion of coaxial bodies [6]:

$$(A(t) - m\rho_C^2(t))\dot{p} + D(t)qr + C_1(t)q\sigma = 0,$$
$$(A(t) - m\rho_C^2(t))\dot{q} - D(t)pr - C_1(t)p\sigma = 0, \quad (2.3)$$
$$C_2\dot{r} + C_1(t)(\dot{r} + \dot{\sigma}) = 0, \quad C_1(t)(\dot{r} + \dot{\sigma}) = M_\delta,$$

where $D(t) = C(t) - A(t)$, $C(t) = C_1(t) + C_2$, $A(t) = A_1(t) + A_2$, and M_δ is the moment of the internal interaction of bodies (torsional moment of engine, the action of friction forces, etc.).

Kinematical equations for the angles of spatial orientation (Fig. 3) have the following form

$$\dot{\gamma} = p\sin\varphi + q\cos\varphi, \quad \dot{\psi} = \frac{1}{\cos\gamma}(p\cos\varphi - q\sin\varphi),$$
$$\dot{\varphi} = r - \frac{\sin\gamma}{\cos\gamma}(p\cos\varphi - q\sin\varphi), \quad \dot{\delta} = \sigma. \quad (2.4)$$

3. APPROXIMATE SOLUTIONS FOR THE ANGLES OF SC ORIENTATION

When making an approximate analysis of motion let us assume that the mass, longitudinal and transverse inertia moments of body *1* (braking engine) decrease according to a linear law, which is correct with a sufficient accuracy for the solid-propellant rocket engines used as brake engines with propellant charges of star-shaped profile section and package-grain charges due to homogeneity of their burning. Let us take the following linear laws of variation of the inertial–mass parameters:

$$m(t) = m_1 + m_2 - \nu t,$$
$$A_1(t) = A_{1,0} - \frac{A_{1,0} - A_{1,k}}{T}t = \alpha(m_1 - \nu t), \qquad (3.1)$$
$$C_1(t) = C_{1,0} - \frac{C_{1,0} - C_{1,k}}{T}t = \beta(m_1 - \nu t),$$

where m_i is the initial mass of the ith body, ν is the mass consumption per second, $A_{i,0}$, $C_{i,0}$, $A_{1,k}$, and $C_{1,k}$ are the values of the equatorial and transverse inertia moments of bodies corresponding to the beginning and end of operation of the brake engine, and T is the time of operation of the brake engine.

The quantities α and β in relations (3.1) represent the coefficients of proportionality linking the values of the moments of inertia of the brake engine with its mass in the process of homogeneous burning of propellant inside the volume. For example, for a cylindrical form of the brake engine the following formulas are valid:

$$C_1(t) = \alpha m_1(t), \quad A_1(t) = \beta m_1(t),$$
$$\alpha = \frac{1}{2}R^2, \quad \beta = \left(\frac{H^2}{12} + \frac{R^2}{4} + z_{C_1}^2\right),$$

where H and R are the height and radius of the brake engine and z_{C_1} is the coordinate of the center of mass of the brake engine in $Ox'y'z'$ coordinate system.

Let there be no interaction between coaxial bodies. In this case it follows from the last two equations of (2.3) that the longitudinal angular velocities are constant: $r = r_0$ and $\sigma = \sigma_0$.

If the propellant burning is uniform, the coordinates of the centers of mass z_{C_i} of individual bodies do not vary, but the coordinate $z_C = z_C(t)$ of the center of mass of the whole system varies. If we take (3.1) into account, the quantity $m\rho_C^2(t)$ in Eqs. (2.3) can be represented in the following form:

$$m\rho_C^2(t) = \frac{[m_2 z_{C_2} + (m_1 - \nu t)z_{C_1}]^2}{m_1 + m_2 - \nu t}$$
$$= \frac{a + b(1-\chi t) + c(1-\chi t)^2}{\kappa + (1-\chi t)}, \qquad (3.2)$$
$$\chi = \nu/m_1, \quad \kappa = m_2/m_1, \quad a = m_1 z_{C_2}^2\kappa^2,$$
$$b = 2m_1\kappa z_{C_2}z_{C_1}, \quad c = m_1 z_{C_1}^2.$$

Since at the initial instant of time the center of mass of the system coincides with the origin of the coordinate system ($z_C(0) = 0$), the following relations are valid:

$$a + b + c = \frac{(m_1 + m_2)^2}{m_1}z_C^2(0) = 0, \qquad (3.3)$$
$$b + 2c = 2mz_{C_i}z_C(0) = 0.$$

Let us introduce new dimensionless variables G and F:

$$p(t) = \omega G(t)\sin F(t), \quad q(t) = \omega G(t)\cos F(t), \qquad (3.4)$$

where $\omega = (r_0(A_{1,0} + A_2 - C_{1,0} - C_2) - C_{1,0}\sigma_0)/(A_{1,0} + A_2)$ is characteristic angular velocity. Variable G in formulas (3.4) represents a dimensionless transverse angular velocity of the system of bodies: $G(t) = (p\sin F + 2q\cos F)/\omega$, and phase F determines the angle between the vector of the transverse angular velocity and the Oy axis.

The first two equations of system (2.3) can be represented in the variables amplitude–phase by means of substitution (3.4):

$$\dot{G} = 0, \quad \dot{F} = \frac{[D_1(1-\chi t) + D_2](\kappa + (1-\chi t))}{a_0 + a_1(1-\chi t) + a_2(1-\chi t)^2}, \qquad (3.5)$$

where $D_1 = m_1(\alpha - \beta)r_0 - \beta m_1\sigma_0$; $D_2 = (A_2 - C_2)r_0$, $a_0 = A_2\kappa - a$; $a_1 = A_2 + \alpha m_1 - b$, $a_2 = \alpha m_1 - c$.

Let us write down the exact solution to system (3.5) for the case when, for example, $a_1^2 - 4a_0a_2 > 0$:

$$G = L_0; \quad F = -\chi^{-1}\left[e_2\tau + \frac{e_1}{2a_2}\ln\left|a_2\tau^2 + a_1\tau + a_0\right| + \left(e_0 - \frac{a_1e_1}{2a_2}\right)\ln|f(\tau)|\right],$$

where

$$e_0 = a_2^{-1}(\kappa D_2 - a_0D_1),$$
$$e_1 = \kappa D_1 + D_2 - D_1a_1/a_2, \quad e_2 = D_1/a_2,$$
$$\tau = 1 - \chi t, \quad f(\tau) = \frac{2a_2\tau + a_1 - \sqrt{a_1^2 - 4a_0a_2}}{2a_2\tau + a_1 + \sqrt{a_1^2 - 4a_0a_2}}.$$

Let us note that, if $a_1^2 - 4a_0a_2 \le 0$, the solution can be represented by inverse trigonometric and homographic functions.

These exact analytical solutions are cumbersome and do not give informative presentation of the motion.

Since the mass of the system decreases, the following condition is valid: $0 \le \chi t < 1$. Expanding exact solutions into series or, what is the same, expanding the right-hand side of the second equation of (3.5) into power series uniformly converging with respect to χt on the interval $\chi t \in [0, 1]$, neglecting the terms of the second and greater orders of smallness, taking (3.3) into account, after integration we arrive at the approximate solutions for the variables amplitude–phase:

$$G = L_0, \quad F(t) = s_0 + \omega t + \mu t^2, \tag{3.6}$$

where

$$\mu = \frac{1}{2}\left(\frac{(A_{1,0} - A_{1,k})k}{T(A_{1,0} + A_2)^2} - \frac{n}{A_{1,0} + A_2}\right);$$

$$k = (A_{1,0} + A_2)\omega; \tag{3.7}$$

$$n = \frac{1}{T}[(A_{1,0} - A_{1,k})r_0 - (C_{1,0} - C_{1,k})(r_0 + \sigma_0)].$$

Solutions (3.6) allow one to find the main characteristics of the rotational motion accurate to the second order, namely, the frequency of rotation ω and a small quadratic correction to phase μt^2.

According to (3.4), equatorial angular velocities are determined by the following relations:

$$\begin{aligned} p(t) &= \omega L_0 \sin(s_0 + [\omega + \mu t]t), \\ q(t) &= \omega L_0 \cos(s_0 + [\omega + \mu t]t). \end{aligned} \tag{3.8}$$

Relations (3.8) generalize formulas obtained in [1] and [6], where the smallness of the value of the final relative displacement of the SC center of mass was assumed, and also in the case when the mass is constant ($\mu = 0$).

Let body *1* (brake engine) rotate rapidly, body *2* being motionless with respect to the longitudinal axis ($r_0 = 0$), and the magnitude of the transverse angular velocity of the system being small in relation to the characteristic angular velocity ω,

$$\varepsilon = \sqrt{p^2 + q^2}/|\omega| = |G| = |L_0| \ll 1. \tag{3.9}$$

We assume the orientation angles γ and ψ to be small ($\gamma = O(\varepsilon)$, $\psi = O(\varepsilon)$). Then the angle of nutation θ (the angle between OZ and Oz_i) is determined by the following approximate formula:

$$\theta^2 \cong \gamma^2 + \psi^2. \tag{3.10}$$

Taking into account relations (3.8), (3.9), and (3.10) let us represent kinematical equations (2.4) in the form

$$\dot{\gamma} = \omega L_0 \cos(F(t) - \varphi), \quad \dot{\psi} = \omega L_0 \sin(F(t) - \varphi),$$

$$\dot{\varphi} = -\gamma\omega L_0 \sin(F(t) - \varphi).$$

By virtue of our assumptions the value of $\dot{\varphi}$ has a higher order of smallness than $\dot{\gamma}$ and $\dot{\psi}$. Therefore, one can assume that on a small time interval of the propellant outburn $\varphi = \text{const} = 0$. Then we can write for the angular velocities $\dot{\gamma}$ and $\dot{\psi}$

$$\dot{\gamma} = \omega L_0 \cos F(t), \quad \dot{\psi} = \omega L_0 \sin F(t). \tag{3.11}$$

There exist two possible cases of the motion realized when the following relations between the quantities hold:

$$1)\ \operatorname{sgn}\omega = \operatorname{sgn}\mu, \quad 2)\ \operatorname{sgn}\omega = -\operatorname{sgn}\mu. \tag{3.12}$$

Assuming for the sake of simplicity that the value of frequency ω is positive, and using Fresnel integrals, for both cases (3.12) one can write the following analytical relations for the angles of orientation of the system (upper and lower signs plus and minus correspond to case 1 and case 2, respectively):

$$\begin{aligned} \psi(t) &= \pm c_{\mp}[S(\lambda(t)) - S(\lambda(0))] \\ &+ s_{\mp}[C(\lambda(t)) - C(\lambda(0))] + \psi_0, \\ \gamma(t) &= c_{\mp}[C(\lambda(t)) - C(\lambda(0))] \\ &\mp S_{\mp}[S(\lambda(t)) - S(\lambda(0))] + \gamma_0, \end{aligned} \tag{3.13}$$

where

$$c_{\pm} = R\cos\left(s_0 \pm \frac{\omega^2}{4\mu}\right), \quad s_{\pm} = R\sin\left[s_0 \pm \frac{\omega^2}{4\mu}\right],$$

$$\lambda(t) = \sqrt{\frac{2|\mu|}{\pi}}\left(t + \frac{\omega}{2\mu}\right), \quad R = L_0\sqrt{\frac{\pi}{2|\mu|}},$$

and

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}x^2\right)dx,$$

and

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}x^2\right)dx$$ are Fresnel integrals.

4. ANALYSIS OF NUTATION MOTION OF SC

It follows from solutions (3.13) that the angles of spatial orientation depend on the combinations of the initial conditions of motion, on initial values of inertia–mass parameters of the system, and on their final variations. When SC is de-orbited, it is necessary to reduce the deviations of the braking thrust vector from a chosen direction, which corresponds to a decrease of the opening angle of the nutation cone (Fig. 1). Let us determine the conditions imposed on SC parameters which guarantee that the value of nutation will decrease in the process of motion of the system with varying mass.

When SC is stabilized by partial spin-up [1], one of the coaxial bodies (body *2*) is not set into rotational motion about its own longitudinal axis, and thus one can assume that $r_0 = 0$ and formula (3.7) takes the form

$$\mu = \omega\frac{\Delta_A C_{1,0} - \Delta_C(A_{1,0} + A_2)}{2C_{1,0}(A_{1,0} + A_2)T}, \tag{4.1}$$

where $\Delta_A = A_{1,0} - A_{1,k}$, and $\Delta_C = C_{1,0} - C_{1,k}$ are positive finite variations of the values of the moments of inertia of body *1* (brake engine).

For the practical problem of decelerating small SC for RSES, conditions (3.9) and (3.10) hold, while inertia–mass parameters of SC are specified by the following ranges of variation in the process of the propellant burnout: $m \sim 65$–50 kg, $A_1 \sim 3$–1 kg m^2, $A_2 \sim 3$ kg m^2, $C_1 \sim 0.4$–0.2 kg m^2, $C_2 \sim 0.3$ kg m^2, and duration T of the deceleration process does not exceed 25 s. These values correspond to the case when the ratios of the final variations of the values of the equatorial and longitudinal moments of inertia of the brake engine to the initial total equatorial moment of inertia of SC and to the initial longitudinal moment of inertia of the brake engine, respectively, are small, and thus the following conditions hold:

$$\Delta_A/(A_{1,0} + A_2) \ll 1, \quad \Delta_C/C_{1,0} \ll 1. \tag{4.2}$$

Let us introduce the instantaneous frequency

$$\Omega(t) = \dot{F} = \omega + 2\mu t = \omega(1 + \tau(t)),$$

$$\tau(t) = \frac{\Delta_A C_{1,0} - \Delta_C(A_{1,0} + A_2)}{4C_{1,0}(A_{1,0} + A_2)T}t.$$

The largest (in absolute value) $\tau(t)$ will be small,

$$\sup_t|\tau(t)| = \frac{1}{4}\left|\frac{\Delta_A}{(A_{1,0} + A_2)} - \frac{\Delta_C}{C_{1,0}}\right| \ll 1.$$

For approximate representation of the motion of the system let us use the method proposed in [7] and consider $\tau(t)$ as a parameter equal to the mean value

$$\bar{\tau} = \frac{1}{8}\left(\frac{\Delta_A}{(A_{1,0} + A_2)} - \frac{\Delta_C}{C_{1,0}}\right). \tag{4.3}$$

Taking (4.3) into account, approximate solutions to Eqs. (3.11) will take the form

$$\psi(t) \approx -\frac{L_0}{1 + \bar{\tau}}[\cos(\bar{\Omega}t + s_0) - \cos s_0] + \psi_0;$$

$$\gamma(t) \approx \frac{L_0}{1 + \bar{\tau}}[\sin(\bar{\Omega}t + s_0) - \sin s_0] + \gamma_0; \tag{4.4}$$

$$\bar{\Omega} = \omega(1 + \bar{\tau}).$$

From expressions (3.10) and (4.4) follows the time dependence of the angle of nutation

$$\theta^2(t) = \frac{2L_0^2}{(1 + \bar{\tau})^2}[1 - \cos(\bar{\Omega}t)]$$

$$+ \frac{2L_0}{1 + \bar{\tau}}\{\gamma_0(\sin(\bar{\Omega}t + s_0) - \sin s_0)$$

$$- \psi_0(\cos(\bar{\Omega}t + s_0) - \cos s_0)\} + \theta_0^2,$$

$$\theta_0^2 = \gamma_0^2 + \psi_0^2.$$

Averaging over the fast phase $\zeta = \bar{\Omega}t$ gives the following approximate formula:

$$\langle\theta^2\rangle = \frac{2L_0^2}{(1 + \bar{\tau})^2} + \frac{2L_0}{1 + \bar{\tau}}d + \theta_0^2, \tag{4.5}$$

where $d = \sqrt{\gamma_0^2 + \psi_0^2}\cos(f + s_0)$, $\sin f = \dfrac{\gamma_0}{\sqrt{\gamma_0^2 + \psi_0^2}}$, $\cos f = \dfrac{\psi}{\sqrt{\gamma_0^2 + \psi_0^2}}$.

Let us consider a particular case when $d = 0$, which is realized when $\gamma_0 = \psi_0$ (in this case $f = \pi/4$) and when $p_0 = q_0$ (in this case $s_0 = \pi/4$), which can always be reached by appropriate choice of the coordinate system. Then it follows from formula (4.5) that in order to diminish the mean angle of nutation it is necessary to increase the sum $1 + \bar{\tau}$, which is equivalent to the following conditions:

$$\bar{\tau} > 0, \quad |\bar{\tau}| \longrightarrow \sup. \tag{4.6}$$

In this case the characteristic frequency ω has the following value

$$\omega = -C_{1,0}\sigma_0/(A_{1,0} + A_2). \tag{4.7}$$

The first condition of (4.6) is equivalent to the inequality

$$\Delta_A/(A_{1,0} + A_2) > \Delta_C/C_{1,0}. \tag{4.8}$$

If the velocity of relative spin-up of the bodies σ_0 is specified, and, as a consequence, at the fixed characteristic frequency ω (4.7), the second condition of (4.6) can be reduced to the condition of increasing absolute value of $\bar{\tau}$ and has the form

$$\Delta_A/(A_{1,0} + A_2) - \Delta_C/C_{1,0} \longrightarrow \sup_{\{\Delta_A, \Delta_C\}}. \tag{4.9}$$

It follows from conditions (4.8) and (4.9) that the final variations of the moments of inertia of the brake engine for the problems of decreasing the cone of nutation during the deceleration of SC, $\{\Delta_A, \Delta_C\}$, are important parameters. In practice these quantities determine the form and inner location of the propellant charges (solid packages of channel burning).

Figure 4 represents a straight line

$$\Delta_A = k\Delta_C, \quad k = (A_{1,0} + A_2)/C_{1,0}, \tag{4.10}$$

determining the boundary of a subset admissible from the point of view of reduction of the mean values of nutation oscillations.

Condition (4.8) holds on the above admissible subset and, consequently, a decrease of the mean values of the angle of nutation takes place.

The points $\{\Delta_A, \Delta_C\}$ located upon straight line (4.10) and most distant from it satisfy condition (4.9). Figure 4 presents a set of points corresponding to the region of possible parameters for SC which are enumerated according to the decrease of the above noted distance, for example, point *1* has the largest positive distance (i.e., it is the best), while point *4* has the smallest negative distance (it has the worst combination of parameters). The estimates obtained allow one to elaborate in actual practice the recommendations related to the location of the solid-propellant packs in the brake engine. It is necessary that the process of propellant burning should lead to such variations of the moments of inertia that, first, the point of parameters $\{\Delta_A, \Delta_C\}$ would be located above the boundary of the admissible subset and, second, would be located at the largest distance from it. For example, this could be achieved by positioning the propellant charges along the longitudinal axis and at the smallest distance to it.

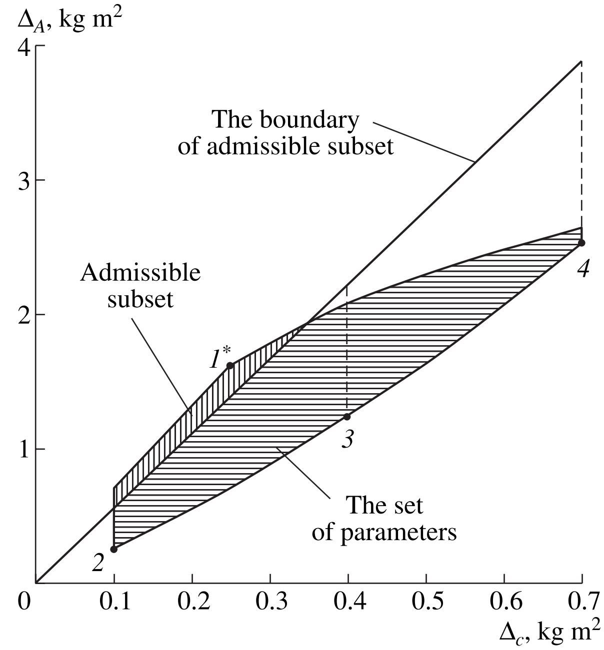

Fig. 4.

5. TRAJECTORY MOTION OF THE CENTER OF MASS AND ESTIMATION OF STABILIZATION EFFICIENCY

The spatial motion of coaxial bodies is characterized by the motion of the longitudinal axis of SC and, consequently, by the direction of the vector of the brake thrust. The efficiency of stabilization is determined by the value of deviation of the final velocity of the SC center of mass on the active leg of trajectory from its known nominal value, which later determines the errors in the initial conditions of transfer to the orbit of descent, in SC enter into the atmosphere, and in the scatter of landing points. In most problems of the dynamics of space flight the decelerating and accelerating impulses are assumed to be instantaneous, and their magnitudes and directions are assumed to be specified [3]. The goal of the study of the trajectory motion of the center of mass on an active leg in this paper is to analyze the process of formation of the brake impulse on a short but finite time interval (taking into account the spatial motion of SC) and also to determine the final velocity of SC center of mass after generation of the impulse.

During the time interval corresponding to a duration of operation of the brake engine, SC in its passive motion (with shut-down brake engine) traverse a part of the initial orbit with a length of ~150 km, assuming that it is circular and has a height of 250 km (the total length of the orbit is 41599.742 km, the velocity of motion along the orbit $V_{orb} = 7.76$ km/s), and the velocity vector of the SC center of mass rotates in the plane of orbit approximately to 1.3°. These remarks allow us to consider with some accuracy the passive motion of the SC center of mass on such an orbital section as uniform and rectilinear motion. If we fix to the SC center of mass the origin of a certain frame of reference $M\xi\eta\zeta$ whose axes keep invariable directions in the absolute space, on the considered orbital section it may be assumed to be inertial, i.e., the gravitational field may be disregarded. Therefore, the motion of the SC center of mass on the active leg in $M\xi\eta\zeta$ coordinate system may be considered as a motion under the action of only a constant (in its value) reactive thrust P. Of course, for a more accurate description of the processes one should take into account noninertiality of the $M\xi\eta\zeta$ coordinate system and thus take into account gravitational forces. Such a refinement presents no difficulties, the more so because the spatial motion of SC studied above in this case does not depend on gravitational forces and all obtained results remain valid. However, there is no necessity to do this in the context of the considered problem of estimating stabilization effectiveness. This problem is to reveal the influence of the SC spatial motion on the process of formation of the brake impulse and on the trajectory motion of the center of mass on the active leg. Thus, at this stage of investigations *gravitational losses* [8] in the increment of the velocity of the center of mass are not taken into account. They can be calculated by independent integration and added to the total increment of the velocity of the center of mass.

Let us consider the motion of the center of mass of coaxial bodies on the active leg of the trajectory of descent with respect to inertial frame of reference $M\xi\eta\zeta$ whose axes $M\zeta$ and $M\eta$ lie in the plane of orbit, while axis $M\xi$ is perpendicular to the orbital plane (Fig. 5). The axis $M\zeta$ corresponds to the calculated direction of the braking impulse which is produced at angle β to the direction of the motion along the orbit ($\beta \sim 55$–45°).

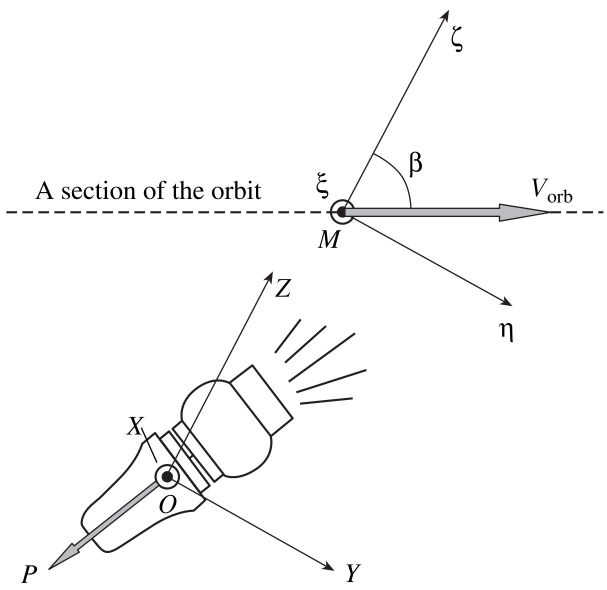

Fig. 5.

The constant force of the jet thrust corresponds to the linear law of mass variation:

$$m(t) = m_0(1-\nu t), \tag{5.1}$$

where $\nu = (m_0 - m_k)/m_0T$.

Let us write the equations of motion of the SC center of mass in projections onto the axes of the $M\xi\eta\zeta$ coordinate system, taking into account the SC spatial motion in the *OXYZ* coordinate system (Figs. 5, 3):

$$\begin{aligned} m(t)\dot{V}_\xi &= -P\sin\gamma, \\ m(t)\dot{V}_\eta &= P\sin\psi\cos\gamma, \\ m(t)\dot{V}_\zeta &= -P\cos\psi\cos\gamma, \end{aligned} \tag{5.2}$$

where $V_\xi = \dot{\xi}$, $V_\eta = \dot{\eta}$, and $V_\zeta = \dot{\zeta}$ are the components of the velocity of the center of mass.

After joint numerical integration of systems of equations (2.3), (2.4), and (5.2) on the time interval *T*, an estimate of efficiency of the stabilization system is carried out. As a rule, in order to do that the following criterion is used:

$$\Pi_1 = \frac{\sqrt{V_{\xi k}^2 + V_{\eta k}^2}}{|V_k|} \le \bar{\Pi}_1, \tag{5.3}$$

where $|\mathbf{V}_k| = \sqrt{V_{\xi k}^2 + V_{\eta k}^2 + V_{\zeta k}^2}$ is the value of the final velocity of the SC center of mass after operation of the brake engine. Estimate (5.3) specifies the "angular" error Π_1 in the generation of the braking impulse corresponding to the deviation of the vector of the final velocity of SC center of mass on the active leg from the direction of $M\zeta$ axis, which is assumed to be the calculated direction of the vector of velocity of the center of mass. Admissible values of errors $\bar{\Pi}_1$ have prescribed values.

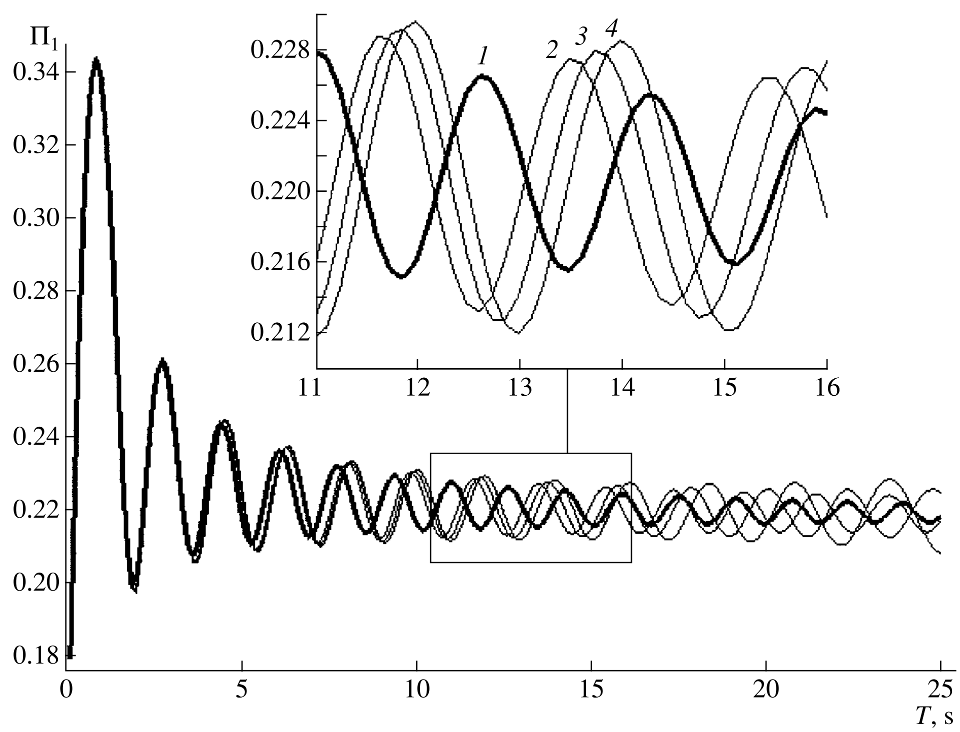

Fig. 6.

Figure 6 presents the plot of the dependence of error Π_1 on the actual duration of the operation of the brake engine; the numerals designate the dependencies corresponding to the points from the region of parameters (Fig. 4), while the bold line presents the dependence corresponding to the best combination of the values—point *1*. It must be noted that the value of error Π_1 is not defined at $T = 0$, i.e., in case of emergency (brake engine is not switched) one cannot speak of the efficiency of its operation. It is also necessary to distinguish the computed duration of operation of the break engine from the actual one, since various nonstandard situations are possible, and thus the errors Π_1 depend on the actual duration of the brake engine operation. The computations were performed for the following parameters of the system and initial conditions of motion: $A_{1,0}$ = 2.5 kg m^2, A_2 = 2.5 kg m^2, $C_{1,0}$ = 0.9 kg m^2, C_2 = 0.3 kg m^2, $m_0 = 65$ kg, $m_k = 50$ kg, $\psi_0 = \gamma_0 = 0.1$ rad, s_0 = 0 rad, r_0 = 0 rad/s, σ_0 = 20 rad/s, p_0 = 0 rad/s, q_0 = 1.1 rad/s, $V_{\xi 0} = V_{\eta 0} = V_{\zeta 0} = 0$ m/s, $T = 25$ s, and P = 1400 N.

It is evident from Fig. 6 that for the best point, all other inertial–mass parameters and initial conditions of SC motion being equal, the smallest mean values of "angular" error are observed, and they decrease when T increases. Thus, above analytical conditions (4.8) and (4.9) of minimization of the values of angles of nutation and, consequently, of the errors of stabilization of the direction of braking impulse are confirmed. Conditions (4.8) and (4.9) determine the form and inner location of the propellant charges; therefore, they should be taken into account when choosing the design parameters of SC.

It should be noted that our results can be useful in the analysis of angular motion of SC of other types when they execute active maneuvers with variation of mass.

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