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CHAOTIC BEHAVIOR OF A PASSIVE SATELLITE DURING TOWING BY A TETHER

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Transport operations in space using tether systems is a most promising technology currently developed in the world. A towing of nonfunctional satellites and orbital stages to the boundary of the atmosphere using tethered space tug relate to such operations. The paper deals with the motion of a tether system, which includes three elements: a passive satellite, a space tug and a viscoelastic tether. Its focus is on the study of attitude motion of the system under the action of a thrust tug and a gravitational moment. The aim of the work is to determine the main features of motion of the tether system depending on its characteristics. The obtained attitude motion equations show that under certain conditions there may be an unstable equilibrium position, which could lead to chaos. In this case, the tether system may unpredictably change your position relative to the line of action of the thrust tug, which coincides with the local horizontal in this task. We have found a condition which excludes the existence of unstable equilibrium position as a source of the chaos. This condition depends on the tether length, mass and thrust of the tug. In this paper we detailed study the case when there is instability. The attitude motion is divided into two classes (disturbed and undisturbed) for small deflection angles of the tether from the local horizontal. The Poincare sections confirm the existence of chaos, as in the case for the small deflection angles of the tether as well as for the general case of the motion. The paper provides guidelines for choosing the parameters of the system (the thrust tug, the mass tug, the tether length and viscoelastic properties of the tether) depending on the mass of the towed body which do not occur chaos. The results of the paper can be useful in the design of the space towing system.

I. INTRODUCTION

At present, a great attention is paid to studying the possibilities of using tethered systems in many space[1-12]. The fundamental paper by Beletsky and Levin[1] has played an important role in providing the basis for the study of the tethered system dynamics. The tethered systems offer numerous ways of beneficial implementation on modern spacecrafts and allow performing multiple tasks including such as removal of a space debris from Earth orbit to the earth's surface. [13-22]. The towing procedure by a tether in an orbit is a relatively new topic. Many aspects of this problem remain unexplored. One of them is the chaotic behavior of tethered satellite systems (TSS), which includes: a space tug, an elastic tether, and a space debris. Chaotic motion of a tethered system has been reported for the first time in a paper by Misra, et al.[8]. Note that it is possible to experience chaotic planar motion in the current paper for nominally circular orbits, because of the existence of the extensibility of the tether and the presence of the thrust, while in the paper[8], chaos can occur only for elliptic orbits or in the presence of both pitch and roll.

This study focuses on attitude motion of two bodies (a passive satellite and an active space tug) connected by viscoelastic tether in a circular orbit under the influence of gravitational moment and thrust force. The low-thrust tug acts along the tangent to the trajectory in the direction opposite to the motion of the two-body tethered system.

We consider separately the influence of these two power factors. If the gravitational moment operates on the system only, then stable position corresponds to the location of the system along the local vertical. On the other hand, if we assume only the action of the horizontal low-thrust tug then the horizontal position of the two-body tethered system will be stable. Clearly, if there are the two power factors, then an unstable equilibrium (a saddle) of the two-body tethered system can be observed an intermediate position between the local vertical and the local horizontal. In fact, we are dealing with the elastic connecting tether and its pitch oscillations perturb a behavior of the system on the whole. The presence of the saddle and the periodic perturbations create preconditions for occurrence of a chaos. The chaos can lead to unpredictable behavior of the system during towing space debris.

The goal is to find the conditions for the existence of the chaos and to illustrate its impact on the behavior of the two-body tethered system on a circular orbit.

II. MATHEMATICAL MODEL

We consider only planar motion of the tethered system in the orbital plane. The tethered system includes an active satellite or (space tug), a viscoelastic tether and a passive satellite an upper stage (space debris) as shown in Fig. 1. The space tug and the space debris are modeled as material points which have masses m_1 and m_2. The tether is weightless. Suppose that an acceleration of the low-thrust tug $w_F = F/(m_1+m_2) \ll g$ is very small, and then the attitude motion of the system can be studied assuming that the orbit remains circular. Taking into account the accepted assumptions the attitude motion equations of the tethered system in the orbital plane can be written as

$$\ddot{\alpha}+\ddot{\theta}+\frac{F}{m_1 l}\sin\alpha-3\omega^2\sin 2\alpha$$
$$+2\frac{\dot{l}}{l}\left(\dot{\alpha}+\dot{\theta}\right)=0, \quad [1]$$
$$\ddot{l}=-\Omega^2\left(l-l_0\right)-2\Omega\zeta\dot{l}+\frac{F}{m_1}\cos\alpha+\dot{\alpha}^2 l \quad [2]$$

where

$$\Omega=\sqrt{\frac{EA}{m_0 l_0}},$$
$$m_0=\frac{m_1 m_2}{m_1+m_2},$$
$$\omega=\sqrt{\mu r^{-3}}$$
$$M_g=3m_0 l^2\omega^2\sin 2\alpha$$

is the gravitational moment, EA is a stiffness of the tether, ζ is damping ratio of the tether, l is length of the tether, l_0 is length of the unstretched tether, μ is the gravitational constant of the Earth, r is orbital radius, α is the angle between the local horizontal and the tether.

For the convenience of analysis, the independent variable can be changed from time to true anomaly $\theta=\omega t$, in which case Eq. [1] can be written as

$$\alpha''+a\sin\alpha+b\sin 2\alpha$$
$$=\Delta a\sin\alpha+2\frac{\Delta'}{(1+\Delta)}\left(\alpha'+1\right) \quad [3]$$

where $(\ )'=d(\ )/d\theta$ is derivative with respect to the true anomaly, $\Delta=\left(l-l_0\right)/l_0$ is tether elongation.

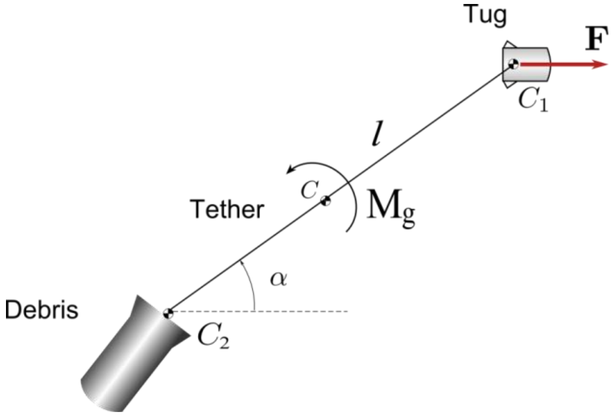

Fig. 1. Space debris and space tug connected by the tether

Assuming that the elongation and derivative of the elongation $\Delta, \Delta' << 1$ are small, Eq. [3] reduces to

$$\alpha''+a\sin\alpha+b\sin 2\alpha=\varepsilon f\left(\theta,\alpha,\alpha'\right) \quad [4]$$

where ε is a small parameter,

$$a=F/\left(m_1 l_0\omega^2\right),\ b=-3, \quad [5]$$
$$\varepsilon f\left(\theta,\alpha,\alpha'\right)=\Delta a\sin\alpha+2\Delta'(1-\Delta)\left(\alpha'+1\right) \quad [6]$$

III. THE EQUILIBRIUM POSITIONS

For the undisturbed motion $(\varepsilon=0)$, Eq. [4] can be further reduced to

$$\alpha''+a\sin\alpha+b\sin 2\alpha=0 \quad [7]$$

or

$$\alpha''=m_\alpha(\alpha)$$

where a biharmonic moment takes the form [Fig. 2 (a)]

$$m_\alpha(\alpha)=-\left(a\sin\alpha+b\sin 2\alpha\right) \quad [8]$$

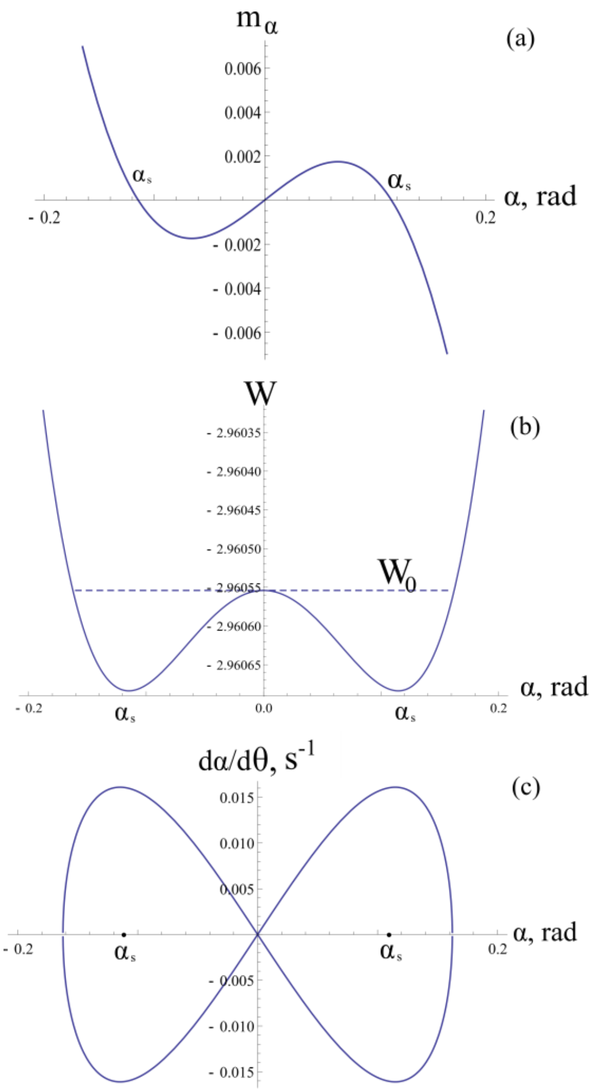

Fig. 2. (a) The biharmonic moment $m_\alpha(\alpha)$,

(b) the potential energy $W(\alpha)$,

(c) the separatrices $S(\alpha)=d\alpha/d\theta$

for $a=5.96055$

Equating to zero the expression [8] leads to two types of stationary solutions

$$\alpha_* = 0 \pm \pi \,,$$

$$\alpha_* = \pm\arccos\left(-\frac{a}{2b}\right) \qquad [9]$$

If

$$a < -2b$$

and taking into account the expressions [5]

$$a = F/\left(m_1 l_0 \omega^2\right) < 6 \qquad [10]$$

then there are the stable equilibrium position

$$\alpha_s = \pm\arccos\left(-\frac{a}{2b}\right) \qquad [11]$$

and the unstable equilibrium position

$$\alpha_u = 0 \qquad [12]$$

It follows from Eq. [5] that the unstable equilibrium could also be for $\alpha_u = \pm\pi$, however, it is impossible for the flexible tether. Note also that if the condition [10] is not satisfied, then the stable position is $\alpha_s = 0$ and the unstable position is absent (Fig. 3).

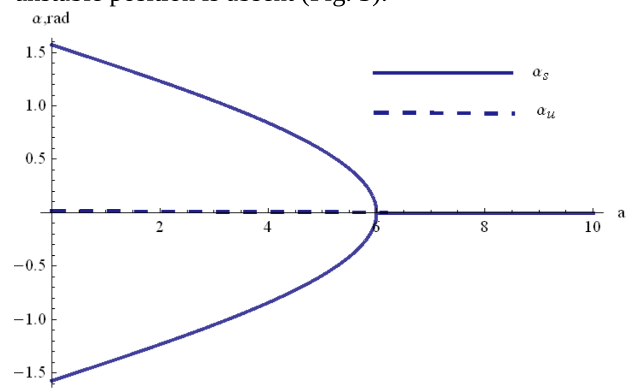

Fig. 3. Bifurcation diagram

IV. LONGITUDINAL OSCILLATIONS OF THE TETHER

For the new variables θ and $\Delta = (l - l_0)/l_0$ Eq. [2] can be re-written as

$$\Delta'' + \sigma^2\Delta + 2\sigma\zeta\Delta' = a\cos\alpha + (1+\Delta)\alpha'^2 \qquad [13]$$

where

$$\sigma = \frac{\Omega}{\omega} = \sqrt{\frac{EAr^3}{m_0 l_0 \mu}} \qquad [14]$$

For small angles of deflection α an approximate solution of Eq. [13] takes the form

$$\Delta(\theta) = A\exp(-\sigma\zeta\theta)\sin(\lambda\theta + \phi_0) + C \qquad [15]$$

where

$\lambda = \sigma\sqrt{1-\zeta^2}$,

$\tan\phi_0 = \dfrac{\lambda}{\sigma\zeta}$,

$A = \dfrac{\Delta_0 - C}{\sin\phi_0}$,

$C = \dfrac{Fm_0}{EAm_1}$

It is obvious that the unstable equilibrium position [12] and the small perturbations [6] caused by longitudinal oscillations [15] of the tether can lead to chaos of the perturbed system [4].

V. POINCARE SECTIONS

In order to study the influences of the small disturbances on the dynamics, the disturbed motion is analyzed by constructing Poincare surfaces in the two-dimensional space $(\alpha, d\alpha / d\theta)$. Constructions of the Poincare surfaces are based on the numerical integration of Eqs. [4] and [13]. All the trajectories shown in Figs. 4-7 start on abscissa axis

$$\alpha_0 \in (-0.4,\ 0.4),\ \alpha_0' = 0$$

Figs. 4-6 present the Poincare sections for the case [10]

$$a = F / \left(m_1 l_0 \omega^2\right) = 5.5 < 2b$$

when there is the unstable equilibrium position (saddle) at the point $\alpha_u = 0$ and there are the stable equilibrium positions (saddle) at the points (center) $\alpha_s = \pm 0.41$. If

$$a = F / \left(m_1 l_0 \omega^2\right) = 6.5 > 2b$$

there is no an unstable equilibrium position and we see stable motion relative to the center $\alpha_s = 0$ (Fig.4).

Modeling of the disturbed motion [Eqs. [3] and [13]] is performed for the following parameters

$\sigma = 198.34$, $\quad \zeta = 0.08$

where for Dyneema $\zeta = 0.08$ and for Kevlar 49 $\zeta = 0.04\text{-}0.08$ [12].

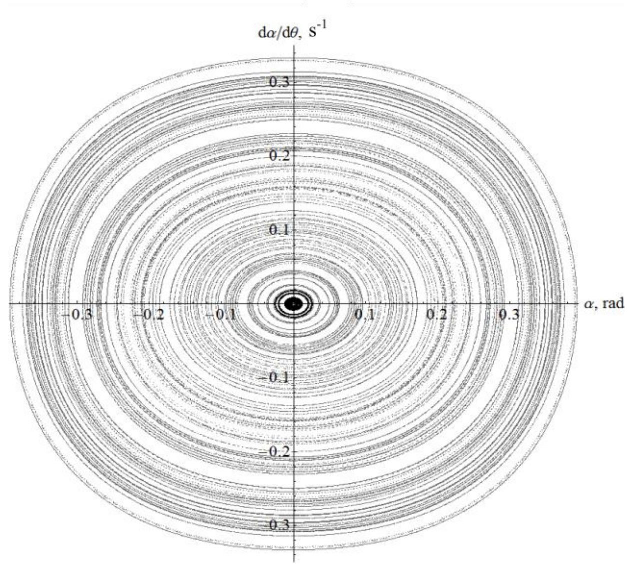

Fig. 4. Poincare sections for disturbed motion for $\zeta = 0.04$ and $a = 6.5$

Fig. 5 shows the Poincare sections for the undisturbed system [7]. We see that the intersection of phase trajectory does not occur in the vicinity of the saddle $\alpha_u = 0$.

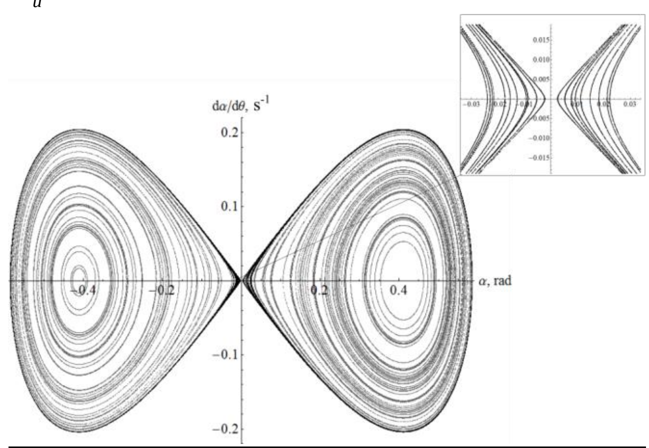

Fig. 5. Poincare sections of undisturbed motion

On the contrary, there is a chaotic intersection for the perturbed motion $\varepsilon \neq 0$ as shown in Fig. 6. Note that the chaotic transitions observed in sufficient proximity to the saddle $\alpha_u = 0$, if the phase paths removed from the saddle, we see stable oscillations about the centers $\alpha_s = \pm 0.41$ (Fig. 6). Thus only the location near the local horizontal of the tether can be considered dangerous when there is the chaotic motion of the tether relative to the local horizontal during towing.

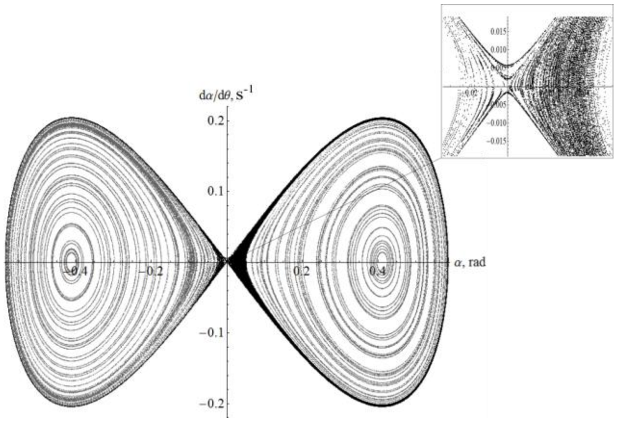

Fig. 6. Poincare sections of disturbed motion for $\zeta = 0.08$ and $a = 5.5$

It became obvious that the elastic oscillations of the tether can cause chaos if there is the unstable equilibrium [12]. Therefore, the choice of thrust and mass of the space tug, and the tether length should be such as to satisfy the condition

$$a = F / \left(m_1 l_0 \omega^2 \right) > 6 \quad [16]$$

when there is not the unstable equilibrium [12].

VI. CONCLUSION

The attitude motion dynamics of the two-body tethered system in circular orbit subject to a gravity gradient torque and a thrust force of a space tug has been studied. The tethered system is perturbed by longitudinal oscillations of a viscoelastic tether. We have investigated numerically the attitude motion dynamics by using construction of Poincare sections. Furthermore, we have found the analytical criteria for determining the characteristics of the tethered system in which chaos does not exist, ie the tether system parameters should be chosen so that the condition [16] is satisfied.

Thus, in considering the dynamics of tethered satellite systems we need to take into account relationship between the librations and elastic oscillations of the tethers. In particular, we have shown that chaos may exist during tethered tow of space debris. The obtained results can be applied to study the possible properties of the space tug and the tether for the space debris removal system.

VI. ACKNOWLEDGMENTS

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REFERENCES

[1] V.V. Beletsky, E.M. Levin, *Dynamics of space tether systems*, Univelt, Univelt Incorporated, 1993. Vol. 83.

[2] E.M. Levin, *Dynamic analysis of space tether missions*, Univelt Incorporated, 2007. Vol. 126.

[3] H. Troger, A.P. Alpatov, V.V. Beletsky, V.I. Dranovskii, V.S. Khoroshilov, and A.V. Pirozhenko, Dynamics of Tethered Space Systems, CRC Press, Boca Raton, FL, 2010.

[4] P. Williams, A. Hyslop, M. Stelzer, and M. Kruijff, “YES2 Optimal Trajectories in Presence of Eccentricity and Aerodynamics Drag,” Acta Astronaut. 64 (7–8), 2009, pp. 745–769.

[5] F. Zimmermann, U. M. Schoettle, E. Messerschmid, “Optimization of the tether-assisted return mission of a guided re-entry capsule”, Aerospace Science and Technology 9, 2005, pp. 713–721.

[6] V.S. Aslanov, A.S. Ledkov, *Dynamics of the Tethered Satellite Systems*, Elsevier, 2012.

[7] A.K. Misra, and V.J. Modi, “A survey on the dynamics and control of tethered satellite systems,” Advances in Astronautical Sciences 62, 1987, pp. 667–719.

[8] A.K. Misra, M.S. Nixon, and V.J. Modi, “Nonlinear dynamics of two-body tethered satellite systems: constant length case," Journal of Astronautical Sciences 49, 2001, pp. 219–236.

[9] K.D. Kumar, and K. Kumar, “Attitude Maneuver of Dual Tethered Satellite Platforms Through Tether Offset Change,” Journal of Spacecraft and Rockets, Vol. 38, No. 2, 2001, pp. 237–242.

[10] P. Williams, C. Blanksby, and P. Trivailo, “Tethered planetary capture maneuvers,” Journal of Spacecraft and Rockets 41, 2004, pp. 603–613.

[11] K.K. Mankala, and S.K. Agrawal, “Dynamic Modeling and Simulationof Impact inTetherNet/Gripper Systems,” Multibody System Dynamics, Vol. 11, No. 5, 2004, pp. 235–250.

[12] M. Kruijff, *Tethers in Space*, Ph.D. Thesis, Delft University of Technology, The Netherlands, 2011.

[13] C. Bonnal, J.-M. Ruault, J.-M., M.-C. Desjean, “Active debris removal: Recent progress and current trends,” Acta Astronaut. 85, 2013, pp. 51–60.

[14] L. Jasper, H. Schaub, “Input shaped large thrust maneuver with a tethered debris object,” Acta Astronaut. 96, 2014, pp. 128–137.

[15] L. Jasper, H. Schaub, “Tethered towing using open-loop input-shaping and discrete thrust levels,” Acta Astronaut. 105, 2014, pp. 373–384.

[16] L. Jasper, Open-loop thrust profile development for tethered towing of large space objects, Ph.D. Thesis, Aerospace Engineering Sciences Department, University of Colorado, Boulder, 2014.
[17] V.S. Aslanov, V.V. Yudintsev, "Dynamics of large debris connected to space tug by a tether," J. Guidance Control Dyn. 36 (6), 2013, pp. 1654–1660.
[18] V.S. Aslanov, V.V. Yudintsev, "Dynamics of large space debris removal using tethered space tug," Acta Astronaut. 91, 2013, pp. 149–156.
[19] V.S. Aslanov, V.V. Yudintsev, "Behaviour of tethered debris with flexible appendages," Acta Astronaut. 104 (1), 2014, pp. 91–98.
[20] V.S. Aslanov, A.S. Ledkov, "Dynamics of towed large space debris taking into account atmospheric disturbance," Acta Mechanica, Vol. 225, Issue 9, 2014, pp. 2685-2697.
[21] V.S. Aslanov, V.V. Yudintsev, "Dynamics, Analytical Solutions and Choice of Parameters for Towed Space Debris with Flexible Appendages," Advances in Space Research 55, 2015, pp. 660-667. doi:10.1016/j.asr.2014.10.034.
[22] V.S. Aslanov, V.V. Yudintsev, The motion of tethered tug-debris system with fuel residuals. Advances in Space Research. Volume 56, Issue 7, 1 October 2015, pp. 1493-1501.