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Tethered towing large space debris with fuel residues by a small spacecraft-tug

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Abstract

The problem of space debris mitigation is one of the most important problems of modern astronautics. Among existing projects and studies, a special place is occupied by an active removal of large space debris, which includes non-functional satellite and upper rocket stages. Many of these space objects have fuel tanks, in which residual fuel can remain. The movement of fuel can have a significant impact on the behavior of the large space debris during its removal. The purpose of this work is to study the dynamics of large space debris with fuel residues during its deorbiting by an active spacecraft-tug. The motion of a mechanical system consisting of a small tug, a massless tether, and passive large space debris with fuel residues is considered in an orbital reference frame. It is supposed that the system is under the influence of the gravitational torques and the constant thrust force generated by engines of the tug. It is assumed that the motion occurs in the plane of the orbit. The tug is considered as a material point, and the space debris is a rigid body. The equations of the space tether system motion were constructed by the means of the Lagrange formalism. The relative equilibrium positions of the system were found. For each of them, the first-approximation equations were constructed. These equations were used to study the influence of the system parameters on the stability of the equilibrium positions. Two different configurations of the space tether system, corresponding to stationary motions, were found as a result of the analysis. Numerical simulation of the mechanical system motion was carried out. Its results were compared with small oscillations, which were determined using the first-approximation equations. The results of the study can be used to select the parameters of the transport system intended for towing and removing large space debris with fuel residues.

Keywords: space debris, towing, tether, fuel residuals, space tug

1. Introduction

Interest in space tethered systems has appeared almost from the beginning of active near-Earth space exploration [1]. The first mention of such systems can be found in the Tsiolkovsky's work of 1895. Tethers were used to ensure the safety of astronauts when they conducted extravehicular activity. The first space experiment with a tether was carried out in 1966 as part of the Gemini-11 mission. At present, the possibility of using space tethered systems for solving various problems in space is widely studied [2-4]. One of the promising application areas of space tethers is active large space debris removal by tethered towing.

Space debris is becoming one of the major challenges of modern astronautics. According to existing studies, it is necessary to remove at least five large non-functioning objects per year in order to preserve the possibility of active use of LEO [5]. Analysis of the large space debris behavior, which is towed by an active spacecraft on a tether, requires studying its motion around the center of mass. The dynamics of such tethered systems are considered in [6-9], taking into account the elastic properties of the tether and space debris, as well as other factors. Existing works do not consider a large class of space debris objects containing fuel on their board. It should be noted that there are studies in which the motion of a spacecraft with fuel remains is investigated [10-12]. This paper is an extension of the studies [13, 14].

The purpose of this work is to study the dynamics of large space debris with fuel residues during its deorbiting by an active spacecraft-tug. The mathematical model of the system will be developed, its equilibrium positions will be found and numerical simulation of the motion of the system will be carried out.

The paper consists of four sections. The state of art and paper purpose are given in Introduction. The second section is devoted to the mathematical model development and to the search its equilibrium positions. The results of numerical simulation and their discussion are given in the third section. The fourth section contains conclusions.

2. Mathematical models and methods

2.1 Equations of motion

Considered mechanical system consists of a small tug, a tether, and passive large space debris with fuel residues (Fig. 1). It is assumed that motion occurs in the orbital plane. The motion of the orbital tug is completely determined by its orientation and propulsion systems, that allows to consider the tug as a material point A of m_1 mass. The thrust force of the tug's engine $\mathbf{F}$ has a constant value, and it is directed along the local horizon. The tether is massless and inextensible rod of l length. The large space debris is rigid body. Its mass is m_2. The principal moments of inertia of the space debris without fuel are I_x, I_y, and I_z. The distance from the point of the tether attachment to the centre of mass of the space debris is denoted by $BC = a$. To describe the motion of the fuel residues in space debris tanks the equivalent pendulum model is used [13]. The mass of the pendulum is m_3, its length is $ED = b$. The distance from the pendulum suspension point to the tether attachment point is $BE = d$.

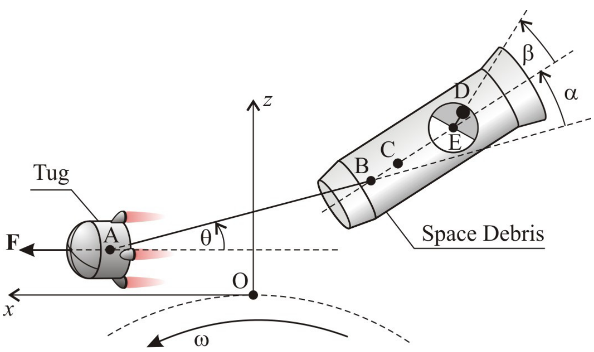

Fig. 1. Space tethered system in orbital reference frame

Equations of the system motion can be written in orbital reference frame $Oxyz$. Its origin moves in a circular orbit and at the initial instant coincides with the position of the tug. The axis Ox lies on the local horizontal and it is directed towards the orbital flight. The axis Oz points away from the Earth centre along the local vertical. The axis Oy is perpendicular to the plane of the orbit and completes the right-hand system. The orbital reference frame rotates around the centre of the Earth with constant angular velocity [15]

$$\omega = \sqrt{\mu r^{-3}}\,, \qquad (1)$$

where μ is the gravitational constant of the Earth, r is the distance from the centre of the Earth to origin O. To obtain the equations of the considered space tethered system motion, the Lagrange equation of the second kind in the matrix form can be used [15]

$$[A(\mathbf{q})]\ddot{\mathbf{q}} + \left[\left[A_{\mathbf{q}}(\mathbf{q})\right]\dot{\mathbf{q}}\right]\dot{\mathbf{q}} - \frac{1}{2}\dot{\mathbf{q}}^T\left[A_{\mathbf{q}}(\mathbf{q})\right]\dot{\mathbf{q}} = \mathbf{Q}\,, \qquad (2)$$

where $[A(\mathbf{q})]$ is the inertia matrix, which elements are given in Appendix A. This matrix is square, positive defined and symmetric. It can be found from the kinetic energy of the system

$$T = \frac{1}{2}\dot{\mathbf{q}}^T\left[A(\mathbf{q})\right]\dot{\mathbf{q}}\,. \qquad (3)$$

the $\mathbf{q} = [x_A, z_A, \theta, \alpha, \beta]^T$ is a generalized coordinate column vector, x_A and z_A are coordinates of the tug in the orbital reference frame, θ is the angle of the tether deflection from Ox axis, α is the angle between the space debris axis and the tether, β is the angle between the pendulum and the space debris axis (Fig. 1). The notation $\left[A_{\mathbf{q}}(\mathbf{q})\right]$ indicates the partial derivative of the inertia matrix with respect the $\mathbf{q}$ vector. The components of the generalized forces vector $\mathbf{Q} = [Q_x, Q_z, Q_\theta, Q_\alpha, Q_\beta]^T$ are determined by the tug's thrust force, gravity force, gravity gradient torque, and inertia forces, which are caused by the rotation of a non-inertial orbital reference frame. These components are given in the Appendix B. In calculating the generalized forces, it was assumed that the quantities $\frac{x_A}{r}$, $\frac{z_A}{r}$, $\frac{l}{r}$ are small. The gravitational and inertial forces were expanded in a series of these small parameters and nonlinear terms were rejected. This technique is widely used for obtaining simplified equations of motion of space tethered systems [1, 4].

2.2 Equilibrium configurations

Various equilibrium configurations can exist in the system. In order to find these configurations, let us equate the generalized velocities to zero in the equations (2).

$$\begin{aligned}
&m\ddot{x}_A = F,\\
&\ddot{z}_A = 0,\\
&F\left(m_2+m_3\right)l\sin\theta = \frac{3\mu m_1\left(m_2+m_3\right)l^2}{2r^3}\sin 2\theta\,,\\
&Fa\left(m_2+2m_3\right)\sin\left(\theta+\alpha\right) =\\
&\quad = \frac{3\mu m\left(I_x - I_z\right)}{r^3}\sin 2\left(\theta+\alpha\right),\\
&\frac{Fm_3 b}{m}\sin\left(\theta+\alpha+\beta\right) = 0;
\end{aligned} \qquad (4)$$

where $m = m_1 + m_2 + m_3$ is the total mass of the system. The first two equations give obvious solutions

$$\ddot{x}_A = \frac{F}{m}, \qquad z_A = const = z_{A0} .$$

From the last three equations of the system (4) two sets of possible values of the angular coordinates for the equilibrium state can be obtained

$$\theta_1 = 0, \alpha_1 = 0, \beta_1 = 0, \tag{5}$$

and

$$\begin{aligned} &\theta_2 = \arccos k_\theta, \quad k_\theta \le 1 \\ &\theta_2 + \alpha_2 = \arccos\left(k_\theta k_m k_I\right), \quad k_\theta k_m k_I \le 1 \\ &\beta_2 = -\left(\theta_2 + \alpha_2\right) \end{aligned} \tag{6}$$

where $k_\theta = \dfrac{Fr^3}{3\mu m_1 l}$ is the coefficient determining the ratio of tug thrust force and forces corresponding to unperturbed orbital motion, $k_m = \dfrac{m_1\left(m_2 + 2m_3\right)}{2m_2 m}$ is the coefficient characterizing the mass ratio of the space tethered system components, $k_I = \dfrac{m_2 a l}{I_x - I_z}$ is the coefficient depending on the distribution of the masses of the large space debris and on the location of the tether attachment point.

In the case when $k_\theta > 1$ and $k_m k_\theta k_I > 1$, there can be only the first configuration of the system (Fig. 2). In order to satisfy the condition $k_\theta > 1$, it is required either to increase the thrust of the tug F, or to reduce the tether length l. To implement the second configuration (Fig. 3), the condition $k_\theta k_m k_I \le 1$ must be satisfied. This condition can be achieved by reducing the distance from the attachment point of the tether to the center of mass of the space debris.

2.3 Analysis of the system motion in the vicinity of the equilibrium positions

To study the oscillations of the system in the vicinity of the equilibrium positions (5) and (6), the equations of the first approximation can be written

$$\left[A_k\right]\ddot{\mathbf{x}} + \left[C_k\right]\mathbf{x} = 0, \qquad k = 1,2 \tag{7}$$

where $\left[A_k\right] = \{a^k{}_{ij}\}$ and $\left[C_k\right] == \{c^k{}_{ij}\}$ is square symmetric matrices of inertia coefficients and elasticity coefficients due to the accelerated motion of the system, $\mathbf{x}$ is the column vector of the deviations of the angles θ, α, and β from the undisturbed motion.

$$\mathbf{x} = [x_\theta, x_\alpha, x_\beta]^T . \tag{8}$$

In the linear configuration, the characteristic equation of the first approximation equation (7) determines three natural frequency of the system

$$k_0\omega^6 + k_1\omega^4 + k_2\omega^2 + k_3 = 0 . \tag{9}$$

For the equilibrium state (5), the coefficients of the characteristic equation (9) can be expressed in terms of the dimensionless parameters: $\eta = m_2/m_1$ is the relative mass of the tug, $\varepsilon = m_3/m_2$ is the relative mass of the fuel, $\lambda = l/a$ is the relative length of the tether, $\delta = d/a$ is the relative length of the pendulum, $\kappa = \left(I_x - I_z\right)/\left(m_2 a^2\right) = l/ak_I$ is the space debris shape factor.

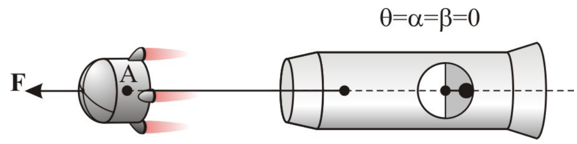

Fig. 2. First equilibrium configuration

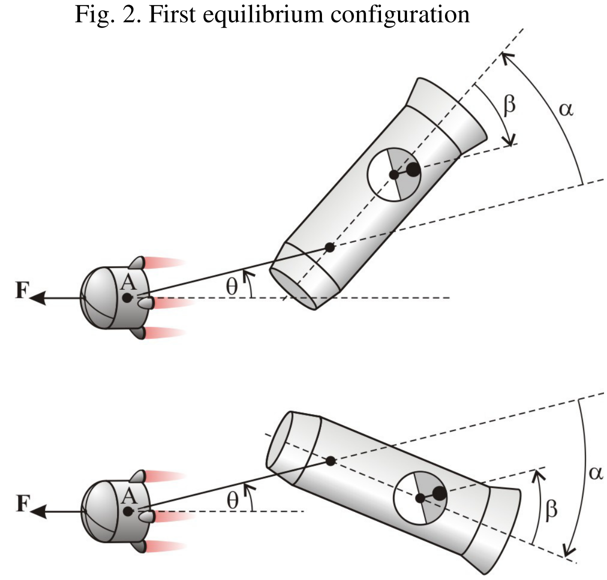

Fig. 3. Second equilibrium configuration

The discriminant of the cubic equation has the form [17]

$$D = -\left(4p^3 + 27q^2\right), \tag{10}$$

where $p = -\dfrac{1}{3}\left(\dfrac{k_2}{k_1}\right)^2 + \dfrac{k_3}{k_1}$, $q = \dfrac{2}{27}\left(\dfrac{k_2}{k_1}\right)^3 - \dfrac{k_2 k_3}{3k_1^2} + \dfrac{k_4}{k_1}$.

The discriminant can take only positive values, since all its roots ω^2 are real. The values of the roots will be closer to each other, the smaller the value of the discriminant.

An analysis of the behavior of discriminant values D with a change in the parameters showed that the minimum values of the discriminant lie above the line

$$\lambda = \kappa\eta . \tag{11}$$

Absolute values of the discriminant decrease with increasing η and λ. The equation (11) gives a critical combination of parameters, in which all three natural frequencies of the space tethered system oscillations can approach one another.

Figure 4 demonstrates the dependence of the natural frequencies on the tether relative length λ. Analysis of equation (11) and Fig. 4 shows that for relatively small values of the tug's relative mass η and the tether relative length λ, it is possible to approach the values of all three natural frequencies ω_i of the system, which can lead to a redistribution of energy between the modes of oscillations.

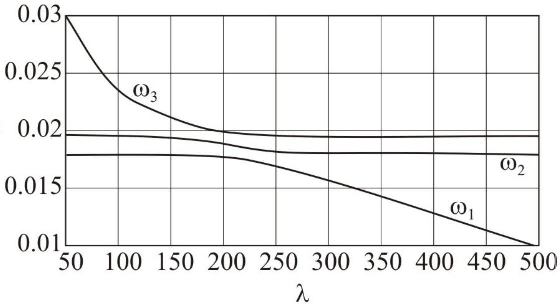

Fig. 4. Dependence of natural frequencies on the tether relative length

In the case of small oscillations of the tethered system around the second equilibrium configuration (6), the coefficients of the characteristic equation depend on more than four parameters. This does not allow to perform a qualitative analysis of the influence of the mechanical system parameters on its behaviour. Numerical analysis of the natural modes and frequencies of the system near the second equilibrium configuration did not detect combinations of the system parameters, for which the discriminant of the corresponding cubic equation (9) tends to zero. This excludes the possibility of convergence of natural frequencies and redistribution of energy between different oscillation modes.

3. Results of numerical simulations

The results of numerical modeling of the oscillations of the system showed that under the same initial conditions, the largest oscillation amplitudes are observed in those cases when all three natural frequencies have close values.

Let us consider the case when the oscillations occur near the first equilibrium configuration (5). The space tethered system has the following parameters: $m_1 = 20\,\text{kg}$, $m_2 = 3000\,\text{kg}$, $m_3 = 30\,\text{kg}$, $r = 6700\,\text{km}$, $F = 1N$, $l = 145m$. As a result of the addition of all three modes of oscillations with close frequencies, beats are observed, under which the energy of oscillations is redistributed between different parts of the system (Fig. 5). The amplitude of the towed space debris oscillations does not remain constant (Fig. 5, angle α), which indicates the transfer of energy to the other elements of mechanical system. The amplitude of the fuel remains oscillations β at certain times is 20 times greater than the amplitude of the oscillations of the space debris α. Although the angle of the tether deflection θ does not exceed α, the addition of oscillations can adversely affect the orientation of the tug. This can create additional difficulties in the operation of the tug orientation system.

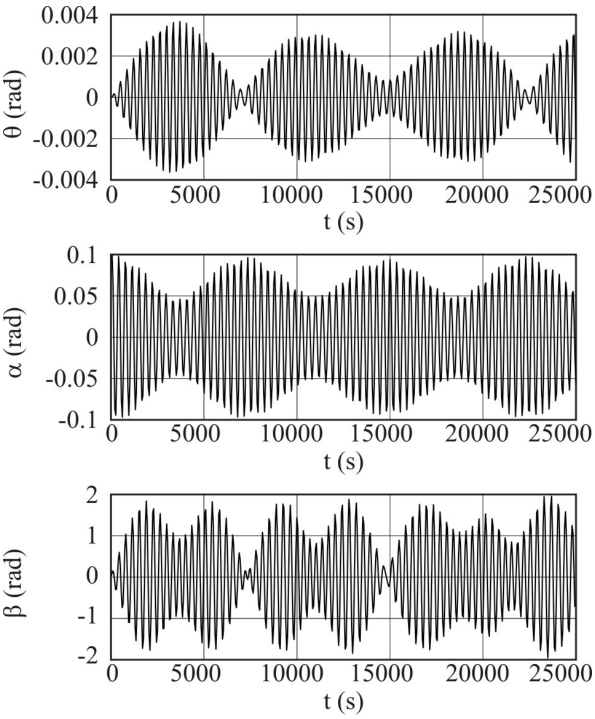

Fig. 5. The change in angles over time with close values of natural frequencies

If the proximity of the natural frequencies is violated, then the regular oscillations of the towed space debris are observed, while the other angles change insignificantly.

Let us consider the case when the oscillations occur near the second equilibrium configuration (6). The space tethered system has the following parameters: $m_1 = 500\,\text{kg}$, $m_2 = 2000\,\text{kg}$, $m_3 = 20\,\text{kg}$, $r = 6700\,\text{km}$, $F = 0.15N$, $l = 100m$. Calculations show that in this case the periods of oscillations are much larger than in the first case. At relatively small initial deviations in the angles θ and α, almost regular oscillations around the equilibrium position are observed. These oscillations are weakly dependent on the fuel residues oscillations, which is convenient to represent in the phase portraits (Fig. 6).

In the case of significant initial deviations, oscillations around both equilibrium configurations occur (Fig. 7). But, as in the case of small perturbations,

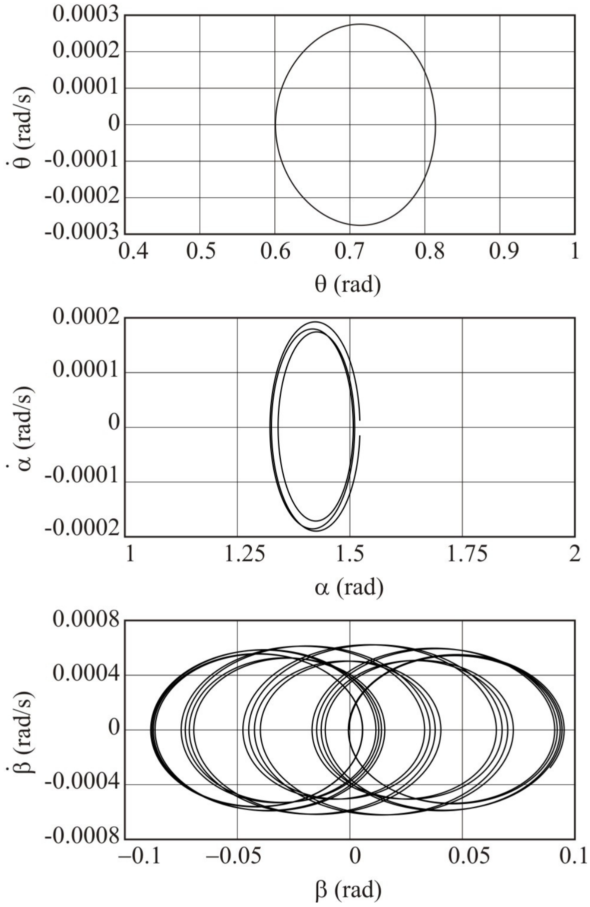

Fig. 6. Phase portraits for small initial deviations

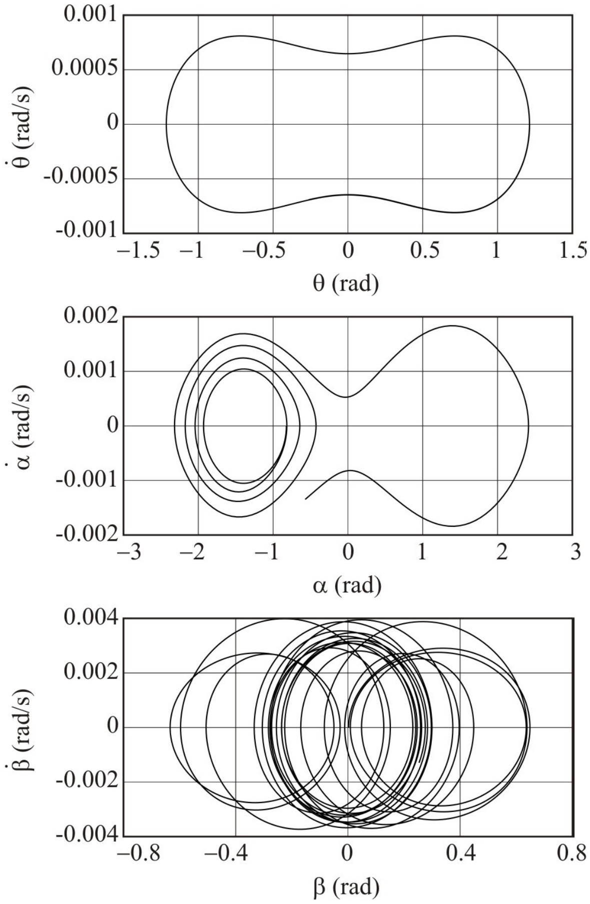

Fig. 7. Phase portraits for large initial deviations

the oscillations of different angles are weakly dependent on each other. It can be argued that in the case of first configuration the presence of fuel residues does not affect noticeably the behavior of the system when the natural frequencies are not close. In the case of second configuration, there is no noticeable effect of fuel residues in tanks at any reasonable ratio of the system parameters (there are no full tanks).

6. Conclusions

In the framework of the study the equations of the plane motion of the space tethered system when towing large space debris with fuel residues in the orbital reference frame were obtained. Using these equations, the equilibrium configurations of the space tethered system and the conditions for their existence were found. The proportions of the system parameters, under which both one and several equilibrium configurations of the space tethered system are possible, were established. An analysis of the parameters influence on the natural frequencies of small oscillations showed that for the first equilibrium configuration the approximation of natural frequencies is possible. The results of numerical simulation confirmed the possibility of increasing the amplitudes of oscillations due to the redistribution of the system energy.

It was found that in the case of the second equilibrium configuration, two types of oscillations are possible. In both cases, the residual fuel does not have a significant effect on the oscillations of the tether and the space debris. For small initial deviations, there are regular oscillations around one of the two possible equilibrium positions. With an increase in initial deviations, the amplitude of the oscillations can be more than twice the value of the corresponding equilibrium position angle.

The results of the study can be used to select the parameters of the transport system intended for towing and removing large space debris with fuel residues.

Acknowledgements

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Appendix A (Elements of the inertia matrix)

The nonzero elements of the inertia matrix $[A(\mathbf{q})]$ are determined through the system parameters and the generalized coordinates as follows

$$a_{11} = m_1 + m_2 + m_3,$$

$$a_{13} = a_{31} = m_2\left[l\sin\theta + a\sin(\alpha+\theta)\right] + \\ + m_3\left[b\sin(\alpha+\beta+\theta) + l\sin\theta + d\sin(\alpha+\theta)\right],$$

$$a_{14} = a_{41} = m_2 a\sin(\alpha+\theta) + \\ + m_3\left[b\sin(\alpha+\beta+\theta) + d\sin(\alpha+\theta)\right],$$

$$a_{15} = a_{51} = m_3 b\sin(\alpha+\beta+\theta),$$

$$a_{22} = m_1 + m_2 + m_3,$$

$$a_{23} = a_{32} = m_2\left[l\cos\theta + a\cos(\alpha+\theta)\right] + \\ + m_3\left[b\cos(\alpha+\beta+\theta) + l\cos\theta + d\cos(\alpha+\theta)\right],$$

$$a_{24} = a_{42} = m_2 a\cos(\alpha+\theta) \\ + m_3\left[b\cos(\alpha+\beta+\theta) + d\cos(\alpha+\theta)\right],$$

$$a_{25} = a_{52} = m_3 b\cos(\alpha+\beta+\theta),$$

$$a_{33} = m_2\left(l^2 + a^2 + 2al\cos\alpha\right) + I_y + \\ + m_3\left[l^2 + d^2 + b^2 + 2bl\cos(\alpha+\beta) + \\ + 2dl\cos\alpha + 2bd\cos\beta\right],$$

$$a_{34} = a_{43} = m_2\left[a^2 + al\cos\alpha\right] + I_y + \\ + m_3\left[d^2 + b^2 + bl\cos(\alpha+\beta) + \\ + dl\cos\alpha + 2bd\cos\beta\right],$$

$$a_{35} = a_{53} = m_3\left(b^2 + bl\cos(\alpha+\beta) + bd\cos\beta\right),$$

$$a_{44} = m_2 a^2 + I_y + m_3\left(d^2 + b^2 + 2bd\cos\beta\right),$$

$$a_{45} = a_{54} = m_3\left(b^2 + bd\cos\beta\right),$$

$$a_{55} = m_3 b^2.$$

Appendix B (Components of the generalized forces vector)

The components of the generalized force vector have the form

$$Q_x = F - 2m_1\omega\dot{z}_A - 2(m_2+m_3)\omega\left(\dot{z}_A + \dot{\theta}l\cos\theta\right),$$

$$Q_z = 2m_1\omega\dot{x}_A + 2(m_2+m_3)\omega\left(\dot{x}_A + \dot{\theta}l\sin\theta\right),$$

$$Q_\theta = \frac{3}{2}\mu\frac{m_1(m_2+m_3)l^2}{(m_1+m_2+m_3)r_O^3}\sin 2\theta + \\ + 2(m_2+m_3)\omega\left(\dot{x}_A + \dot{\theta}l\sin\theta\right)l\cos\theta - \\ - 2(m_2+m_3)\omega\left(\dot{z}_A + \dot{\theta}l\sin\theta\right)l\sin\theta,$$

$$Q_\alpha = \frac{3}{2}\mu\frac{(I_x - I_z)}{r^3}\sin 2(\theta+\alpha),$$

$$Q_\beta = 0.$$

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