Chaotic Behavior of a Passive Satellite During Towing by a Tether

Vladimir S. Aslanov
aslanov_vs@mail.ru   http://aslanov.ssau.ru

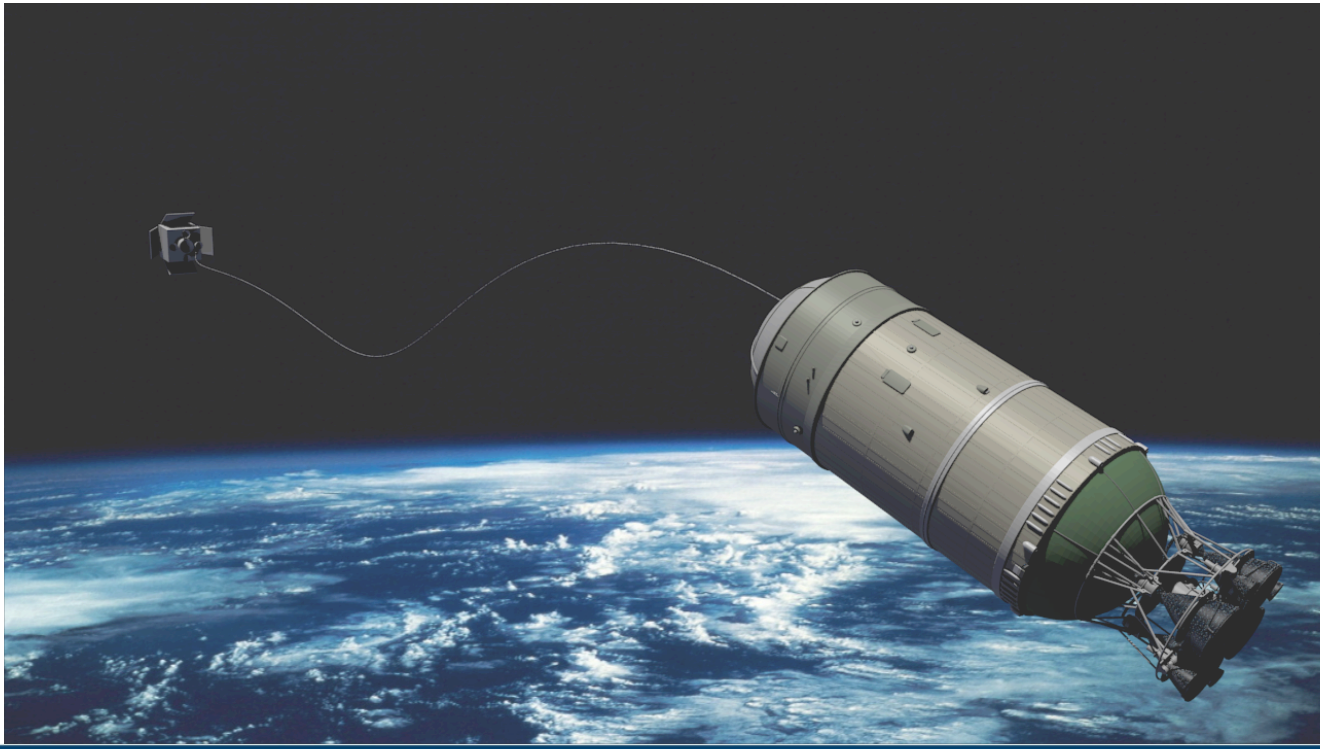
Evgeniy V. Aslanov
aslanov.ev@gmail.com

Samara State Aerospace University, Russia

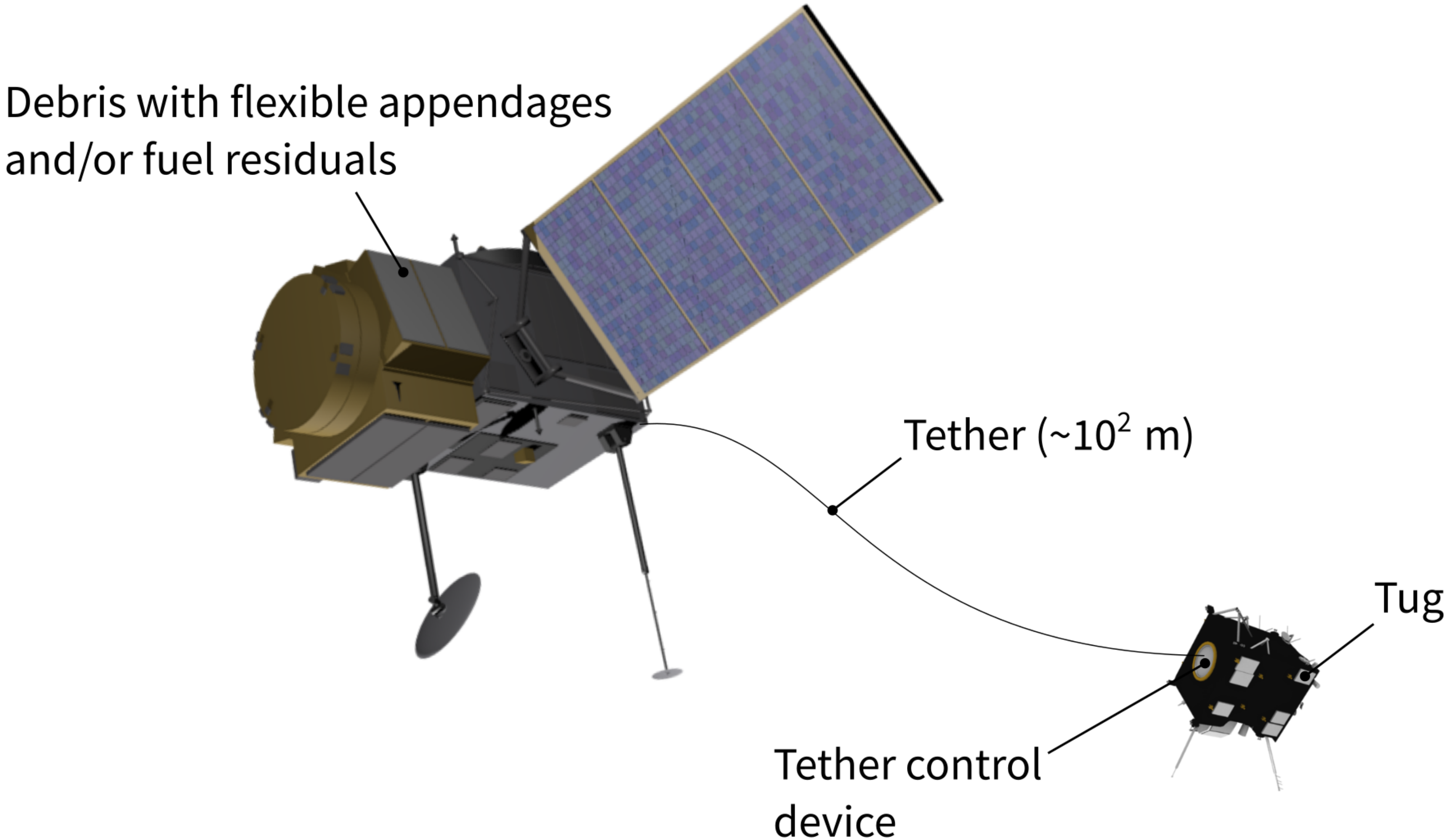
IAC 2015, Jerusalem, 12-16 October 2015

Outline

1. Introduction
2. Mathematical model
3. The Equilibrium position
4. Chaos
5. Poincare sections
6. Conclusion

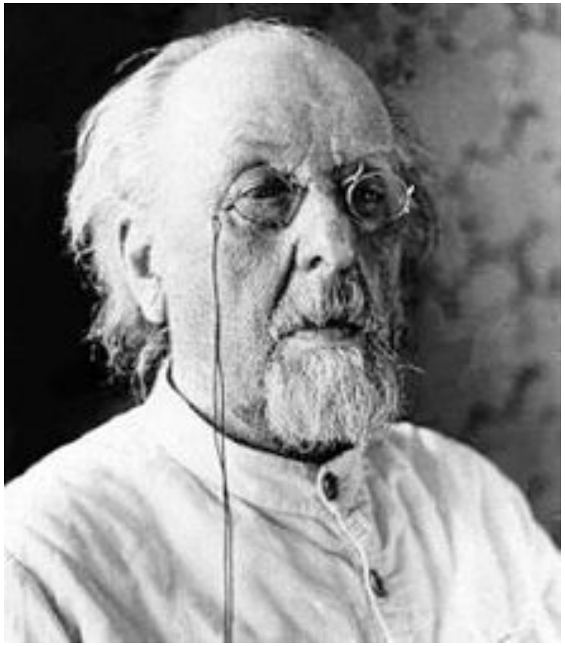


Tug-Tether-Debris system

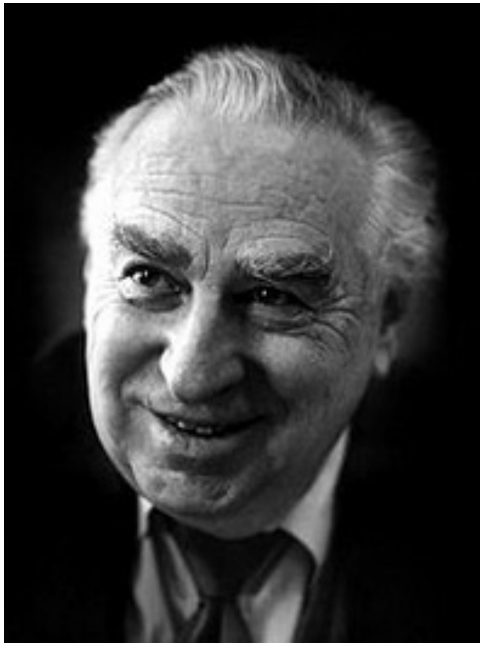


Space tether systems

Space tether system (STS) – mechanical system of rigid bodies moving in different orbits, and the tethers (cables, ropes) that connect these bodies.



Konstantin Tsiolkovsky

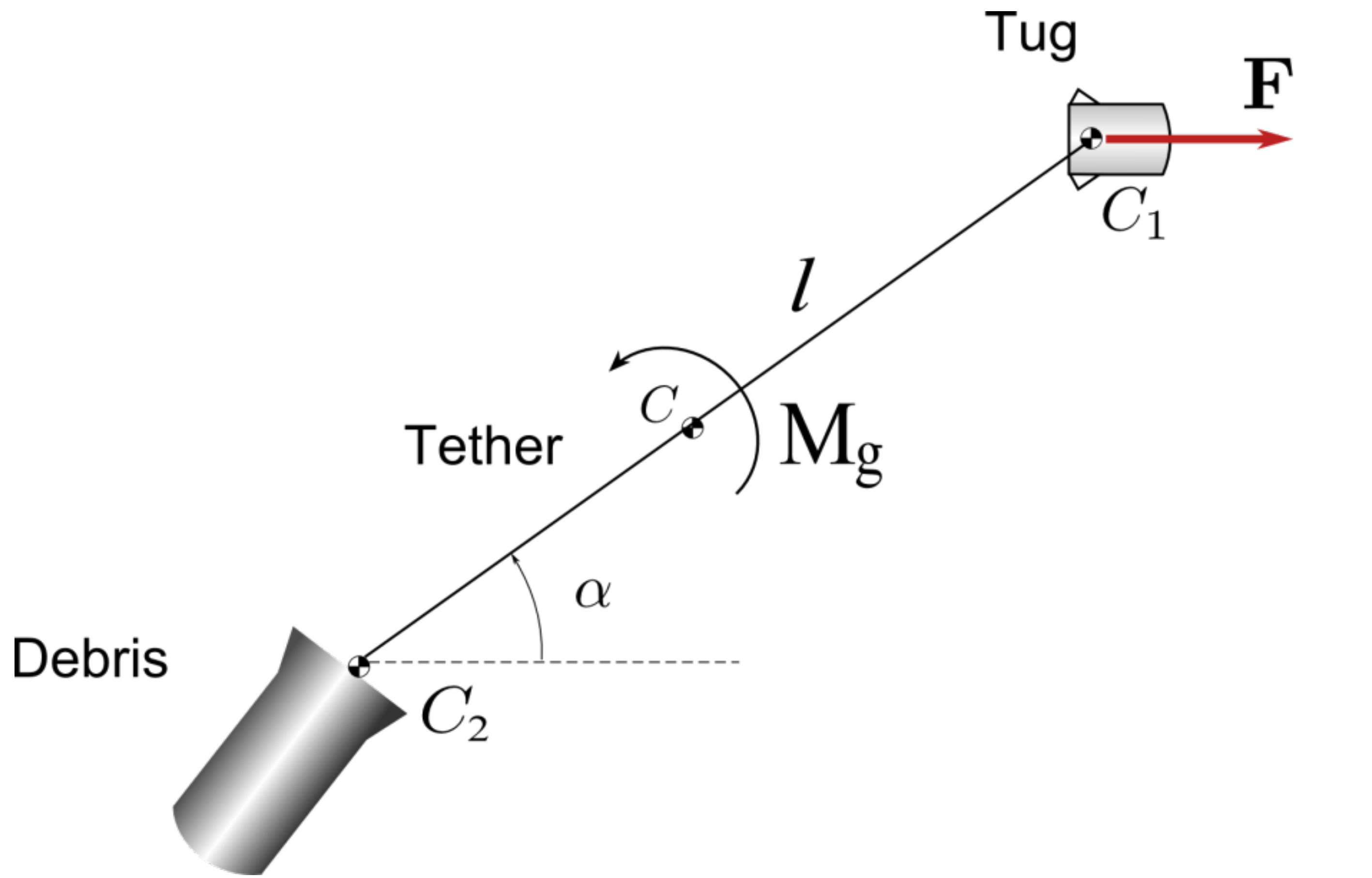


Vladimir Beletskii

Dynamics of STS has been studied by: Beletsky V. V., Levin E. M., Cartmell M.P., Cosmo M.L. , Lorenzini E.C., Misra A.K., Modi. V.J., Williams P., Kruijff M. , Fujii H. A., Edwards B. C., Kumar K. D., Kumar R., McCoy J. E., Sorensen K., Zimmermann F. et al.

Mechanical scheme

The system includes a space tug ***C_1***, a viscoelastic tether ***l*** and a space debris ***C_2***



$M_g = 3m_0 l^2 \omega^2 \sin 2\alpha$ is gravitational moment

F is thrust

The basic assumptions

1. An acceleration of the low-thrust tug is very small

$$w_F = F/(m_1 + m_2) = g$$

then we can study an attitude motion of the system assuming that the orbit remains circular

2. The tether is weightless

$$m_t = 0$$

3. Tether elongation and derivative of the elongation are small

$$\Delta = (l - l_0)/l_0 << 1,\ \Delta' = d\Delta/d\theta << 1$$

where θ is true anomaly

Mathematical model

Equation of the transverse oscillations of the tether

$$\alpha'' + a\sin\alpha + b\sin 2\alpha = \Delta a \sin\alpha + 2\frac{\Delta'}{(1+\Delta)}(\alpha' + 1) = \varepsilon f(\theta, \alpha, \alpha') \quad (1)$$

Equation of the longitudinal oscillations of the tether

$$\Delta'' + \sigma^2\Delta + 2\sigma\zeta\Delta' = a\cos\alpha + (1+\Delta)\alpha'^2 \quad (2)$$

where $(\ )' = d(\ )/d\theta$

$$a = F/(m_1 l_0 \omega^2),\ b = 3,\ \omega = \sqrt{\mu r^{-3}},\ \Omega = \sqrt{EA/m_0 l_0},$$

$$m_0 = m_1 m_2/(m_1 + m_2),\ \sigma = \Omega/\omega = \sqrt{EAr^3/(m_0 l_0 \mu)},\ \varepsilon << 1$$

where l_0 is length of the unstretched tether, r is orbital radius, EA is a stiffness of the tether, ζ is damping ratio of the tether

The Equilibrium Positions

For the undisturbed motion $(\varepsilon = 0)$

$$\alpha'' - m_\alpha(\alpha) = 0 \quad \left[\, m_\alpha(\alpha) = -(a\sin\alpha + b\sin 2\alpha) \,\right]$$

Bifurcation diagram

If $a = F/\left(m_1 l_0 \omega^2\right) < 6$ then

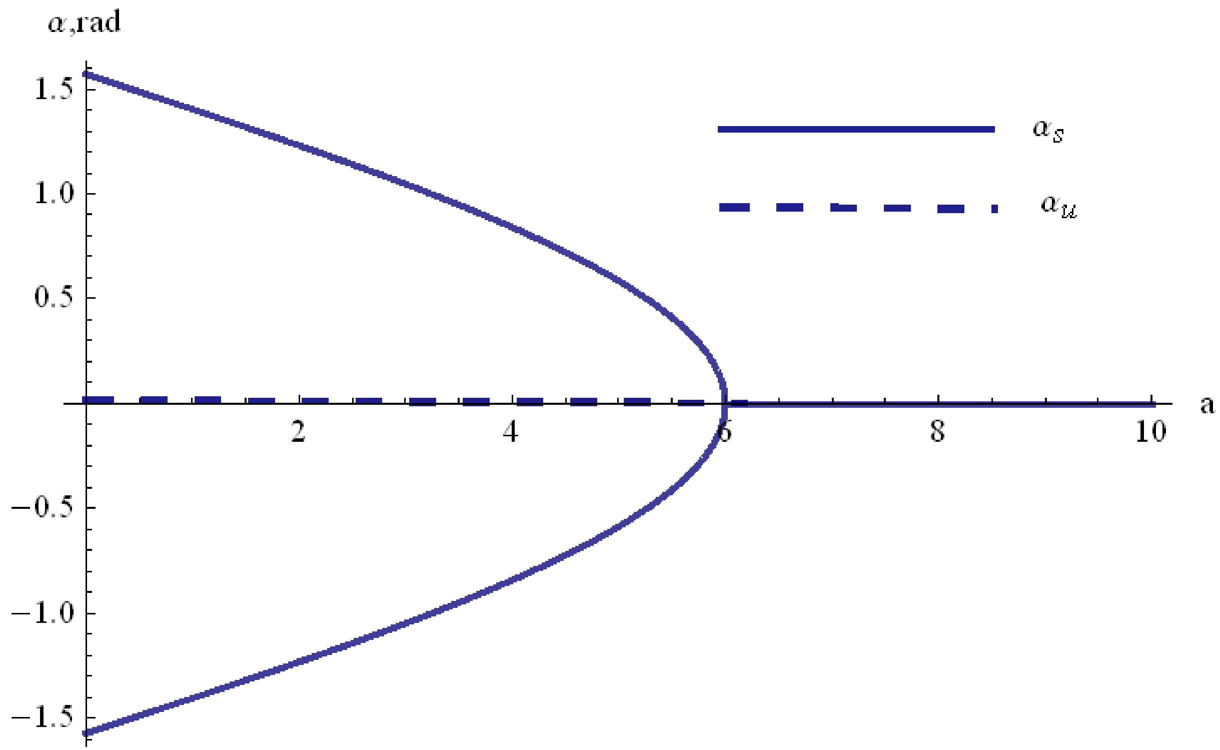
$$\alpha_s = \pm\arccos\left(-\frac{a}{2b}\right)$$

$$\alpha_u = 0$$

If $a = F/\left(m_1 l_0 \omega^2\right) > 6$ then

$$\alpha_s = 0$$

and the unstable equilibrium position does not exist



The Equilibrium Positions

The biharmonic moment

$$m_\alpha(\alpha) = -(a\sin\alpha + b\sin 2\alpha)$$

The potential energy

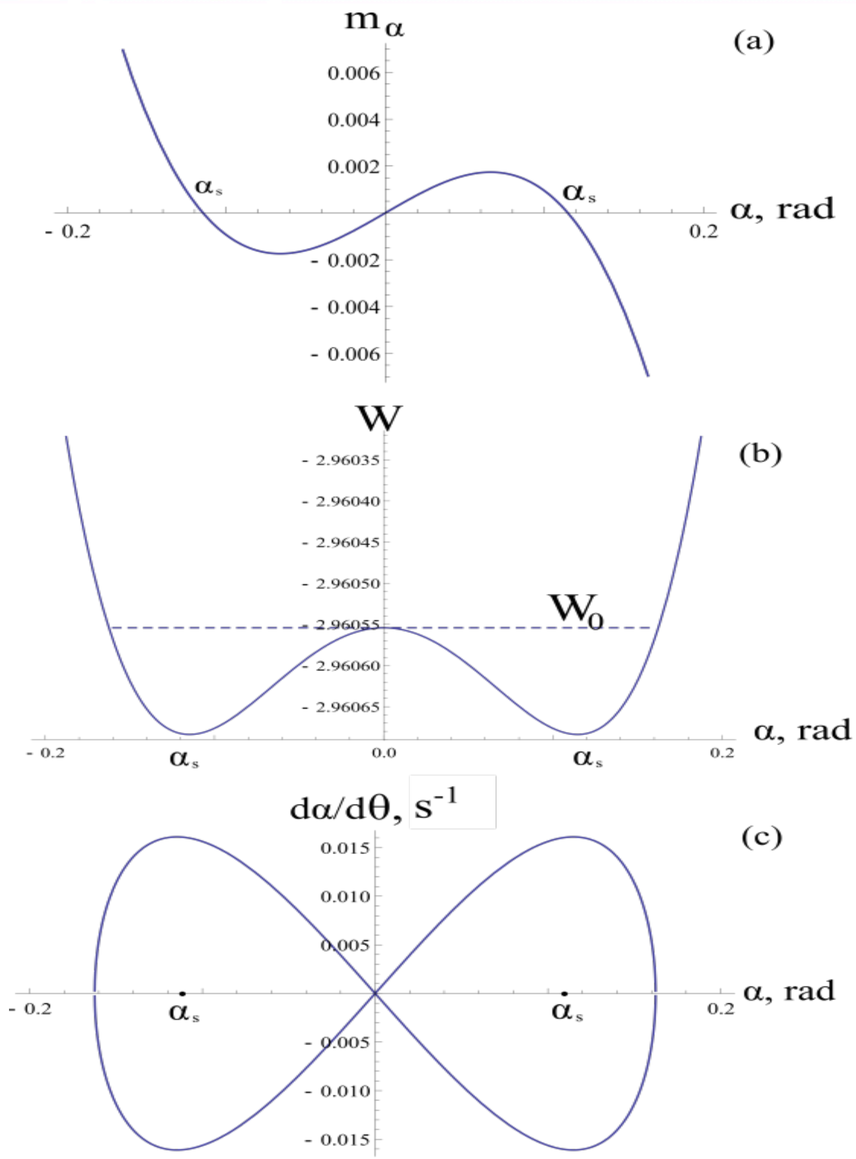
$$W(\alpha) = -\int m_\alpha(\alpha)d\alpha$$

$$= -a\cos\alpha - b\cos^2\alpha$$

The separatrices

$$d\alpha / d\theta = S(\alpha)$$

for $a = 5.96055,\ b = -3$



Chaos

An approximate solution of Eq. (2) takes the form

$$\Delta(\theta) = A\exp(-\sigma\zeta\theta)\sin(\lambda\theta + \phi_0) + C$$

Equation (1) describes the perturbed motion

$$\alpha'' + a\sin\alpha + b\sin 2\alpha = \Delta a\sin\alpha + 2\frac{\Delta'}{(1+\Delta)}(\alpha' + 1)$$

and if

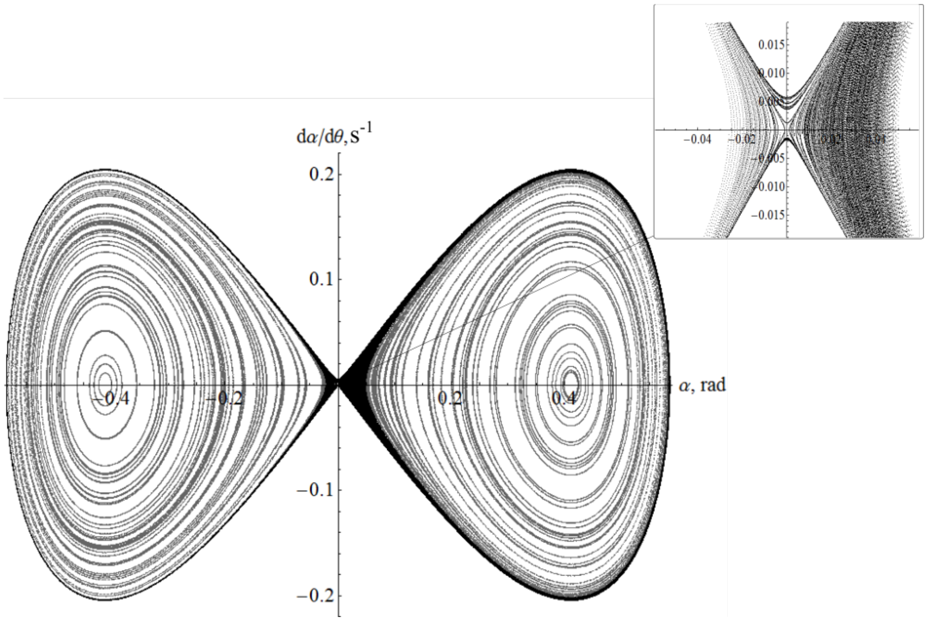
$$a = F/\left(m_1 l_0 \omega^2\right) < -2b = 6$$

Then chaos becomes possible

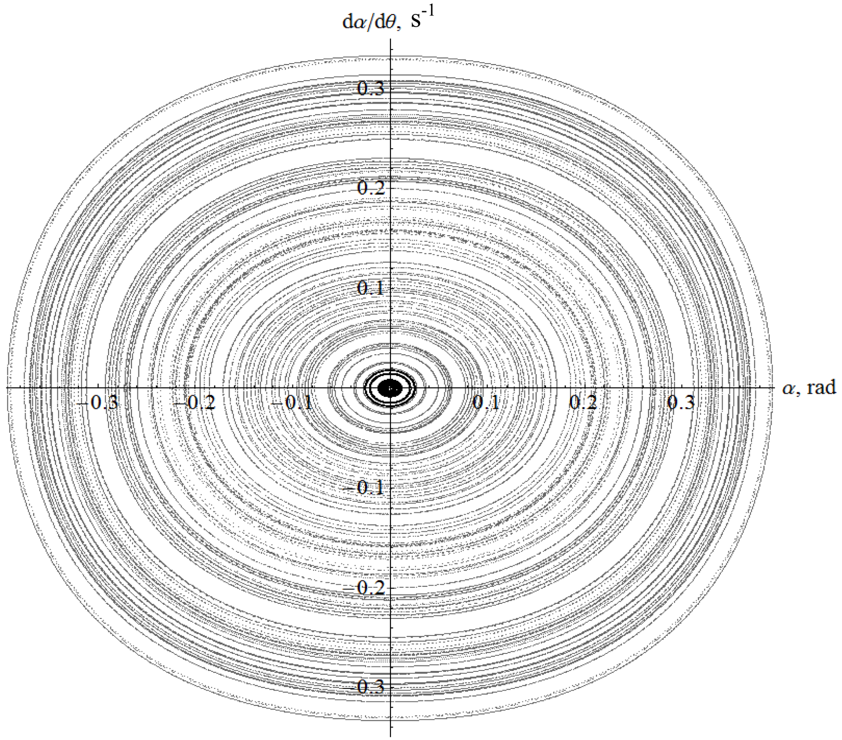
Poincare sections

Poincaré surfaces of section are used to reveal the existence of a stochastic layer in the presence of damping of the tether

$$\zeta = 0.04$$



$$a = F / \left(m_1 l_0 \omega^2\right) = 5.5 < 2b$$



$$a = F / \left(m_1 l_0 \omega^2\right) = 5.5 < 2b$$

Conclusion

The results of this study suggest that a chaos can be observed for the motion of the tethered tug-debris system caused by longitudinal oscillations of the viscoelastic tether

We have studied numerically the attitude motion dynamics by using construction of Poincare sections.

Furthermore, we have found the analytical criteria for determining the characteristics of the tethered system in which chaos does not exist

Thank you

Vladimir S. Aslanov
aslanov_vs@mail.ru http://aslanov.ssau.ru

Evgeniy V. Aslanov
aslanov.ev@gmail.com